UNIVERSITÀ
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DI PADOVA

A FEM-Peridynamics coupling approach for the simulation of dynamic non-linear multi-physics problems involving crack propagation

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PhD course in **Sciences, Technologies and Measurements for Space (STSM)**

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1. Introduction

Problem
Thermo-mechanical phenomena causing damage in aerospace structures

Current solution
Scheduled inspections and maintenance, with consequent increase of costs

Proposed solution
Development of a reliable numerical tool to improve safety and limit financial losses

Peridynamics

Non-local continuum theory with an integral formulation capable of modeling discontinuities in the displacement field, such as crack initiation and evolution of any type.

Equation of motion of point x

$$\rho(\mathbf{x})\,\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathcal{H}_\mathbf{x}} \mathbf{f}(\mathbf{x},\mathbf{x}',t)\,\mathrm{d}V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t)$$

ρ : density

$\mathbf{u}$: displacement vector field

δ : horizon

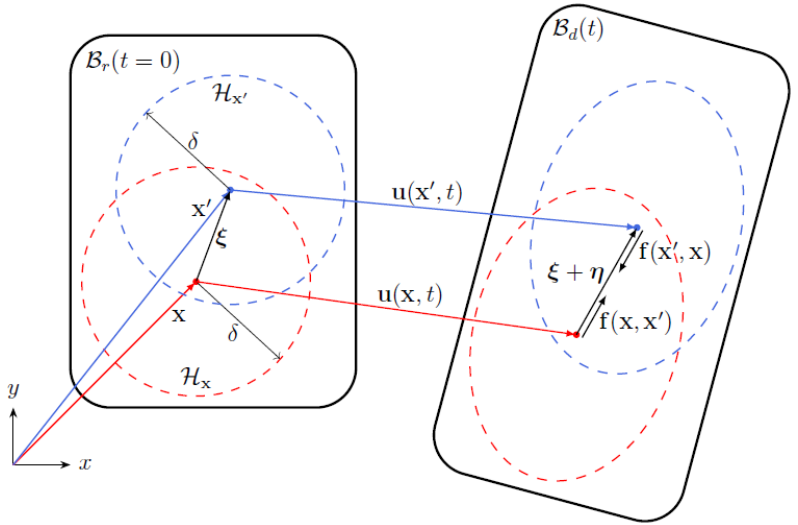
$\mathbf{b}$: body force vector

$\boldsymbol{\xi} = \mathbf{x}' - \mathbf{x}$: relative position vector (bond)

$\boldsymbol{\eta} = \mathbf{u}(\mathbf{x}',t) - \mathbf{u}(\mathbf{x},t)$: relative displacement vector

$\mathbf{f}(\mathbf{x},\mathbf{x}',t)$: pairwise force density

$\mathcal{H}_\mathbf{x}$: neighborhood



Reference and deformed configuration of a peridynamic body Ω: each point x interacts with the points x' in its neighborhood H_x through the bonds.

2. Research background

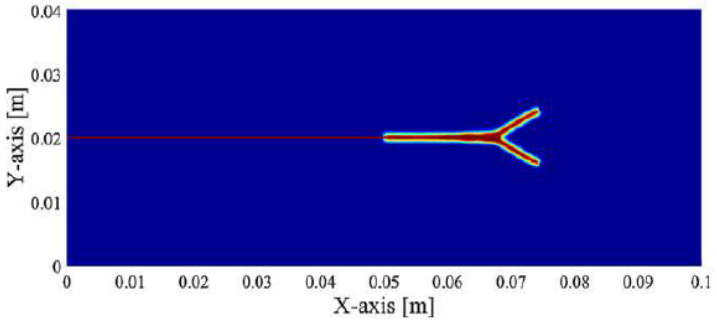
Peridynamics

Main advantages

1. Introduction of the concept of structural damage for a material point
2. No ad-hoc criteria to model crack initiation and propagation

Disadvantages

1. Higher computational cost than classical continuum models
2. Accurate quadrature is required
3. Issues on mechanical properties near the boundary (surface effect) and on the imposition of boundary conditions



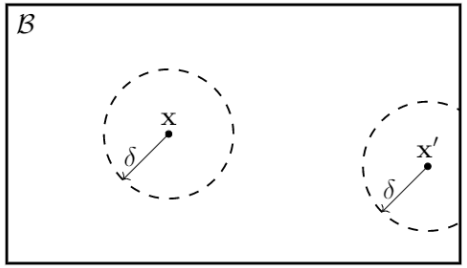
Simulation of a crack propagation in a pre-notched specimen modeled with Peridynamics.

3. Project objectives

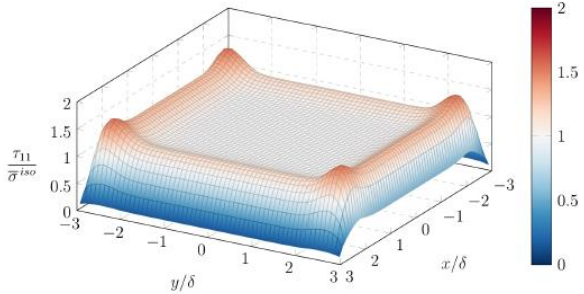
1. Development of an innovative method to mitigate boundary problems due to peridynamic non-locality
2. Improvement of currently used algorithms to compute quadrature weights
3. Coupling of Peridynamics with local models to reduce the overall computational cost
4. Implementation of the FEM-Peridynamics multi-physics coupling for thermo-mechanical problems

4. Results of the first-year work

Mitigation of boundary problems

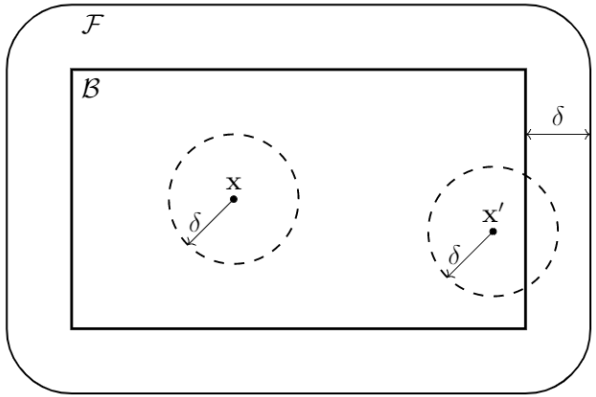


Peridynamic body without boundary corrections.



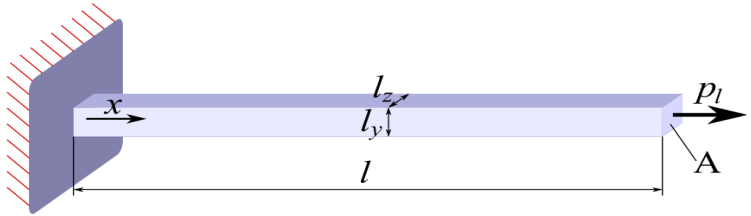
Stiffness fluctuation (surface effect) near the boundary due to the incomplete neighborhoods.

A novel method is proposed in order to mitigate the surface effect and properly impose the non-local boundary conditions in Peridynamics.

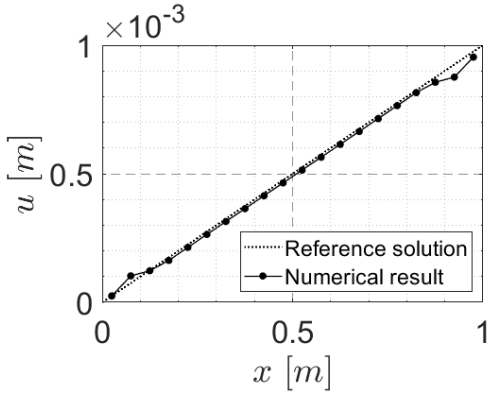


4. Results of the first-year work

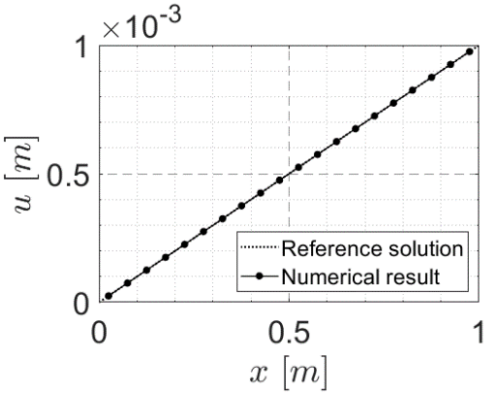
Mitigation of boundary problems



1-dimensional example.



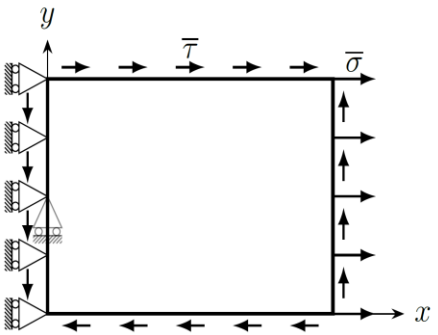
Solution without correction.



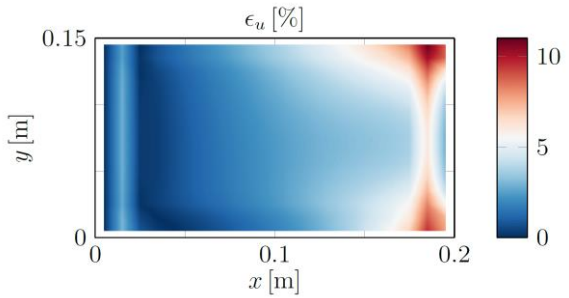
Solution with the proposed method.

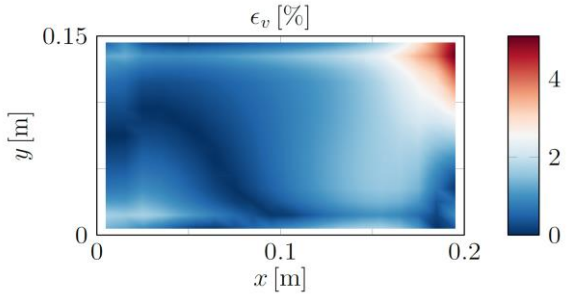
4. Results of the first-year work

Mitigation of boundary problems

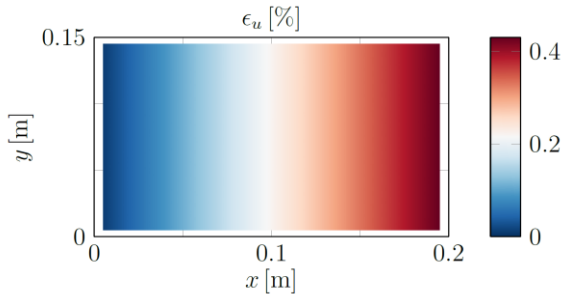


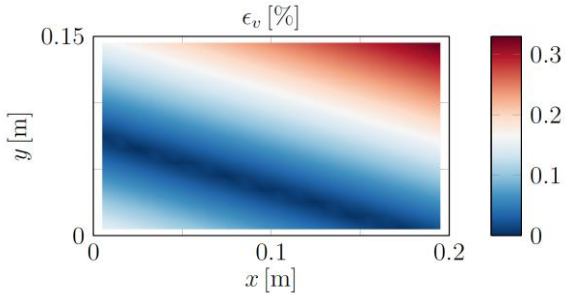
2-dimensional example.





Errors of the solution without correction.



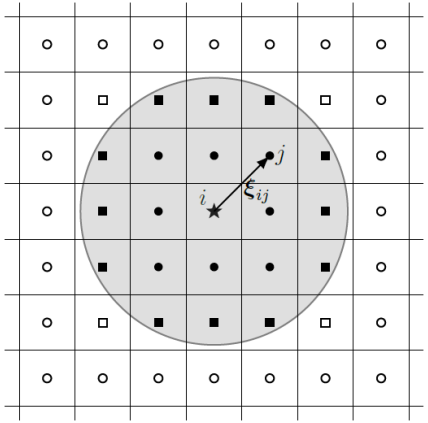


Errors of the solution with the proposed method.

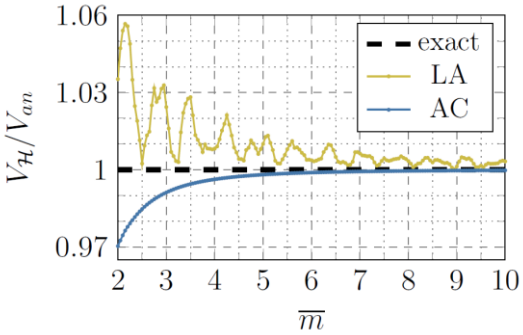
4. Results of the first-year work

New algorithm for peridynamic quadrature

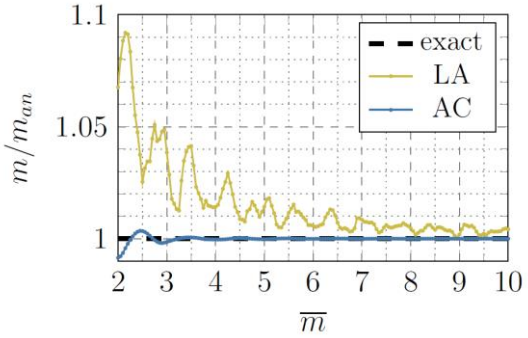
An algorithm to compute accurately quadrature weights in 3D peridynamic models is developed.



Discretization of the neighborhood (2D model for image clarity).



Normalized volume of the neighborhood.

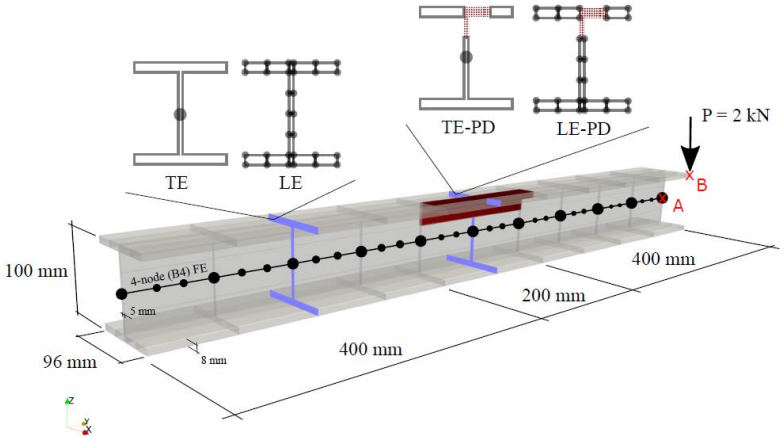


Normalized weighted volume.

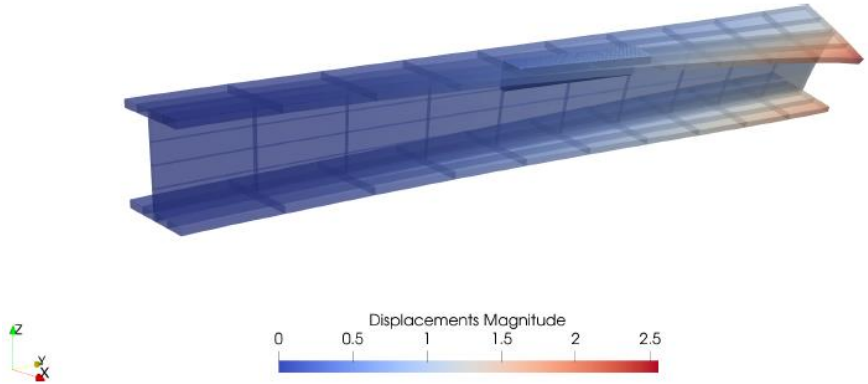
4. Results of the first-year work

Coupling of Peridynamics with local models

Peridynamics is coupled with Carrera Unified Formulation (CUF), a local theory able to reproduce arbitrary kinematic models, to reduce the overall computational cost.



Coupled model.

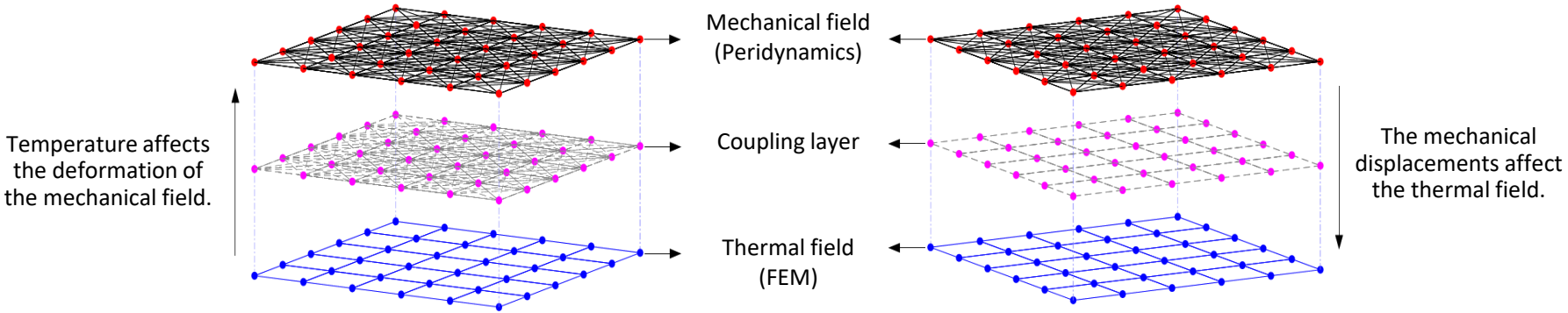


Numerical results, in good accordance with the results of a refined 3D FEM model.

5. Future developments

FEM-Peridynamics multi-physics coupling

Peridynamic theory is employed to model the mechanical field since it is suitable to accurately simulate the initiation and propagation of cracks, whereas FEM is employed to model the thermal and electrical fields since it reduces the computational time of the simulation.



Peridynamic formulation of the mechanical problem.

FEM formulation of the thermal problem.

5. Research schedule

Gantt chart

WBS NUMBER	TASK TITLE	% OF TASK COMPLETE	FIRST YEAR												SECOND YEAR												THIRD YEAR											
			T1			T2			T3			T4			T1			T2			T3			T4			T1			T2			T3			T4		
			O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S
1	**Mitigation of boundary problems in Peridynamics**																																					
1.1	Bibliographic research	100%																																				
1.2	Formulation of the Taylor-based extrapolation method in 1D	100%																																				
1.3	Extension of the Taylor-based extrapolation method to 2D and 3D models	42%																																				
2	**New algorithm for numerical quadrature in Peridynamics**																																					
2.1	Bibliographic research	67%																																				
2.2	Algorithm for numerical quadrature in 2D models	67%																																				
2.3	Algorithm for numerical quadrature in 3D models	33%																																				
3	**Coupling of Peridynamics with classical mechanics**																																					
3.1	Coupling of Peridynamics with CUF	13%																																				
3.2	Improvements on FEM-Peridynamics mechanical coupling	0%																																				
4	**Implementation of FEM-Peridynamics physical coupling**																																					
4.1	Bibliographic research	0%																																				
4.2	Implementation of FEM-Peridynamics physical coupling for thermo-mechanics	0%																																				
5	**Educational activities**																																					
5.1	Exams	89%																																				
5.2	Conferences and seminars	38%																																				
6	**Writing of papers/conference proceedings and PhD thesis**																																					
6.1	Writing of papers/conference proceedings	24%																																				
6.2	Writing of PhD thesis	0%																																				

6. Final remarks

1. An innovative method to mitigate boundary problems in 1D and 2D peridynamic models has been implemented. This method will be extended to 3D models.
2. A new algorithm to compute quadrature weights in 3D peridynamic models has been developed. The results obtained with this algorithm will be thoroughly assessed.
3. Peridynamics has been coupled with CUF to reduce the overall computational cost. The spurious forces arising at the interfaces will be decreased by improving the coupling technique.
4. The FEM-Peridynamics multi-physics coupling will be implemented for thermo-mechanical problems.

Thanks for the attention