Università degli Studi di Padova

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Multiphysics modelling of thermal cracks in multiphase heterogenous porous materials

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PhD Course in Sciences, Technologies And Measurements For Space

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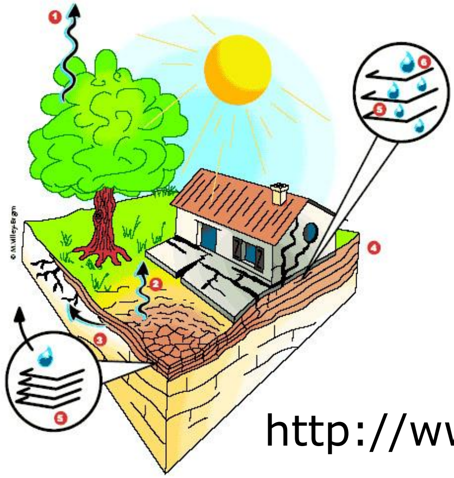
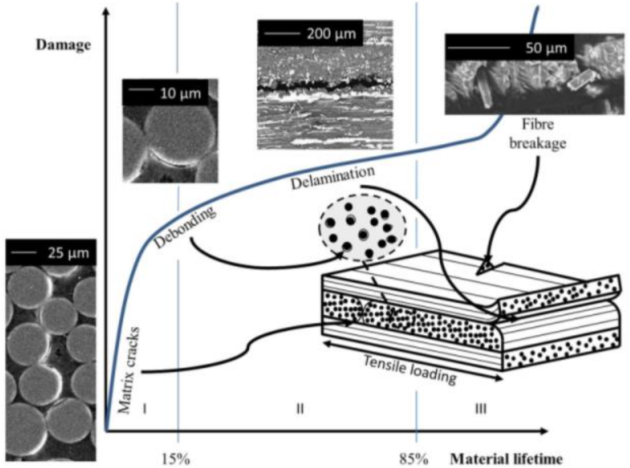
Introduction

Motivation: cracks due to hydro-thermal effects

Desiccation (Laloui 2009)



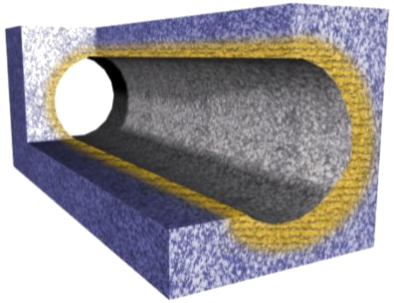
http://www.argiles.fr/

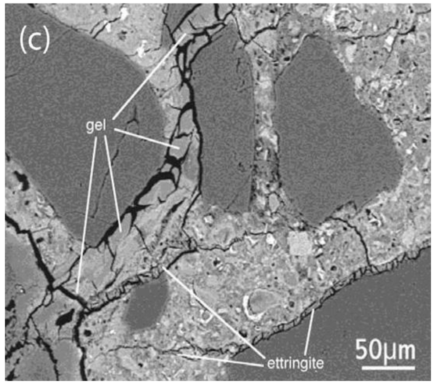


Damage in composite laminates doi.org/10.1007/s10853-018-2045-6

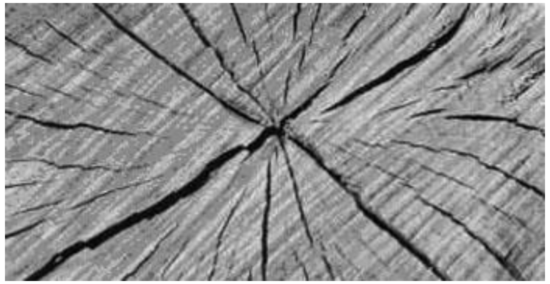
Desaturation/cracks developement of EDZ (Excavation Damaged Zone)

Deep nuclear waste disposals





ASR (Alkali-Silica Reaction) shrinkage-induced cracking in concrete



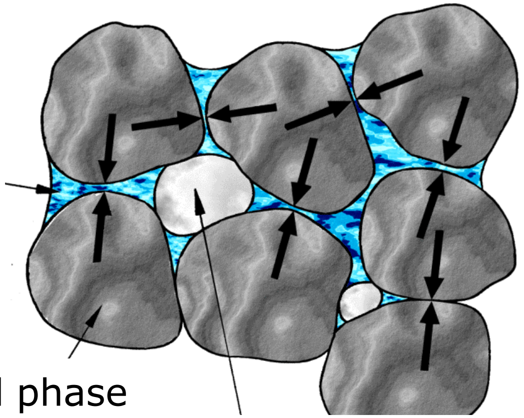
Cracks in wood

Research background

Multi-phase porous material

Composed by a solid skeleton with open pores containing one or more fluids

liquid phase: water

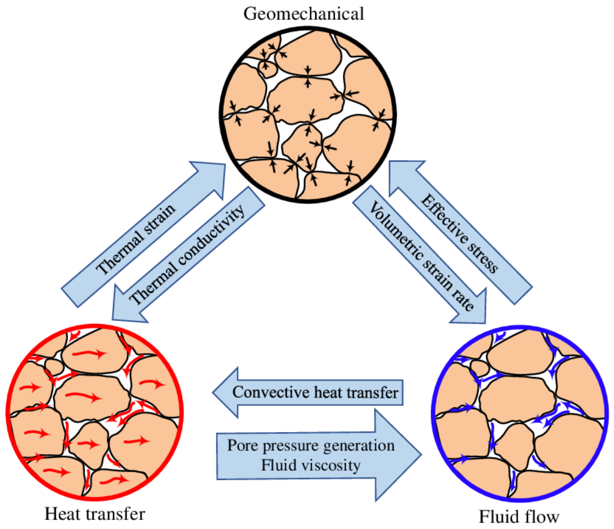


Microscopic view of a three-phase material

Intergranular forces due to capillary effects

solid phase

gas phase: dry air and water vapour



Mechanics of multi-phase porous materials

solid-fluids interaction, liquid-gas interaction, non-isothermal conditions

- *Lewis, R. W., Schrefler, B. A. 1998. The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media (Second.). Chichester, UK: John Wiley & Sons.*
- *William G. Gray, Cass T. Miller 2014. Introduction to the Thermodynamically Constrained Averaging Theory for Porous Medium Systems. Springer.*

Multi-phase porous material

Equilibrium equations (mixture; quasi-statics):

$$div\left(\boldsymbol{\sigma}' - \left[p^g - S_w p^c\right]\mathbf{1}\right) + \rho\mathbf{g} = 0$$

Mass balance equation (solid, liquid water and water vapour):

$$n\left[\rho^w - \rho^{gw}\right]\left[\frac{\partial S_w}{\partial t}\right] + \left[\rho^w S_w - \rho^{gw}\left[1 - S_w\right]\right] div\left(\frac{\partial \mathbf{u}}{\partial t}\right) + \left[1 - S_w\right] n\left[\frac{\partial \rho^{gw}}{\partial t}\right]$$

$$-div\left(\rho^g \frac{M_a M_w}{M_g^2} \mathbf{D}_g^{gw} grad\left(\frac{\partial p^{gw}}{\partial p^g}\right)\right) + div\left(\rho^w \frac{\mathbf{k}k^{rw}}{\mu^w}\left[-grad(p^g) + grad(p^c) + \rho^w \mathbf{g}\right]\right)$$

$$+div\left(\rho^{gw} \frac{\mathbf{k}k^{rg}}{\mu^g}\left[-grad(p^g) + \rho^g \mathbf{g}\right]\right) - \beta_{swg}\frac{\partial T}{\partial t} = 0$$

- *Lewis, R. W., Schrefler, B. A. 1998. The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media (Second.). Chichester, UK: John Wiley & Sons.*

Research background

Multi-phase porous material

Dry air mass balance equation:

$$-n\rho^{ga}\left[\frac{\partial S_w}{\partial t}\right]+\rho^{ga}[1-S_w]div\left(\frac{\partial \mathbf{u}}{\partial t}\right)+nS_g\frac{\partial \rho^{ga}}{\partial t}-div\left(\rho^g\frac{M_aM_w}{M_g^2}\mathbf{D}_g^{ga}grad\left(\frac{p^{ga}}{p^g}\right)\right)$$

$$+div\left(\rho^{ga}\frac{\mathbf{k}k^{rg}}{\mu^g}\left[-grad(p^g)+\rho^g\mathbf{g}\right]\right)-[1-n]\beta_{swg}\rho^{ga}[1-S_w]\frac{\partial T}{\partial t}=0$$

Energy balance equation (mixture):

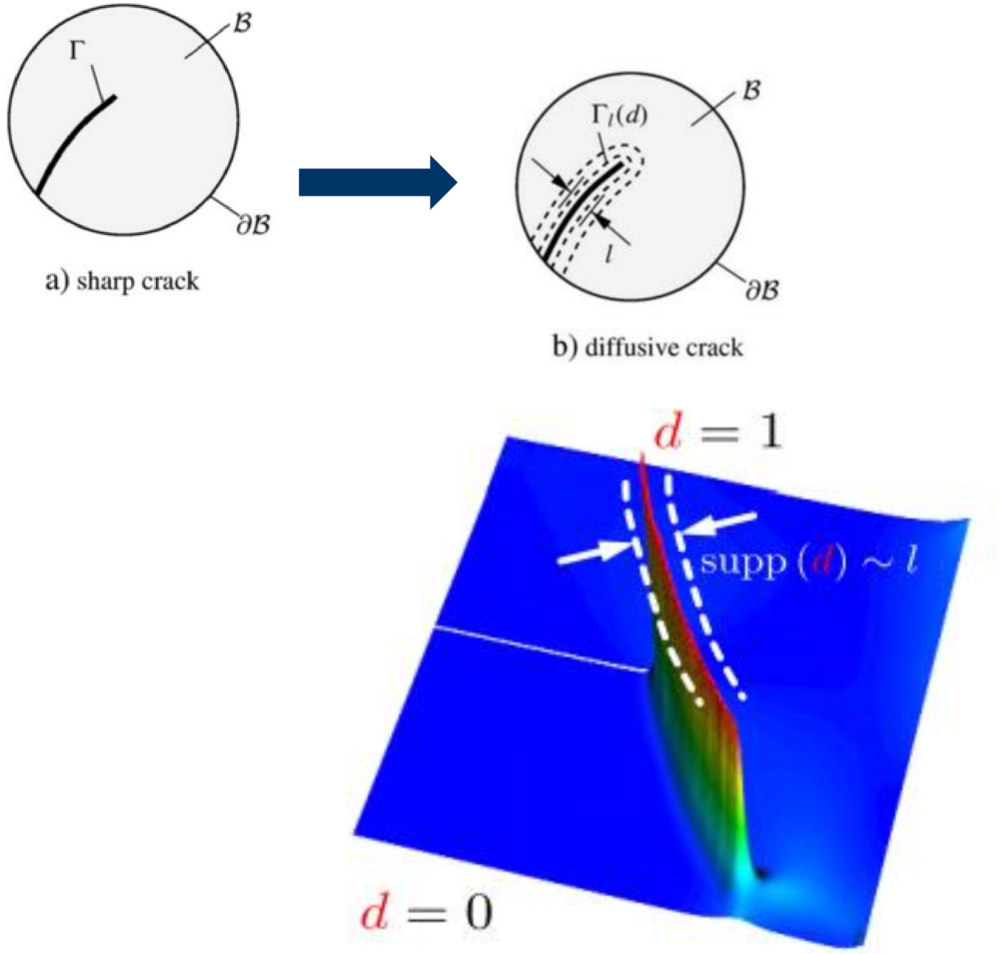
$$\left(\rho C_p\right)_{eff}\frac{\partial T}{\partial t}+\rho^wC_p^w\left(\frac{\mathbf{k}k^{rw}}{\mu^w}\left[-grad(p^g)+grad(p^c)+\rho^w\mathbf{g}\right]\right)\cdot grad(T)$$

$$+\rho^gC_p^g\left(\frac{\mathbf{k}k^{rg}}{\mu^g}\left[-grad(p^g)+\rho^g\mathbf{g}\right]\right)\cdot grad(T)-div\left(\chi_{eff}\,grad(T)\right)=-\dot{m}_{vap}\,\Delta H_{vap}$$

- *Lewis, R. W., Schrefler, B. A. 1998. The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media (Second.). Chichester, UK: John Wiley & Sons.*

Research background

The Phase-Field Method (PFM) to fracture



Key advantages

- Flexibility (initiation, propagation, merging, branching)
- Variational framework
- Simple implementation

Disadvantages

- Fine mesh needed
- Efficiency/robustness of solution

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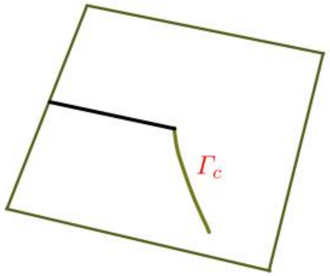
The Phase-Field Method (PFM) to fracture

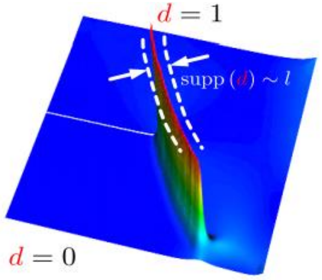
Sharp crack: (variational reformulation of) Griffith

$$\min_{\mathbf{u}\in\mathcal{U}_n(\Gamma_c),\Gamma_c\supseteq\Gamma_{cn-1}} \mathcal{E}(\mathbf{u},\Gamma_c) = \underbrace{\int_{\Omega\backslash\Gamma_c} \psi(\varepsilon(\mathbf{u}))d\Omega}_{\text{elastic strain energy}} + \underbrace{G_c\mathbb{H}^{d-1}(\Gamma_c)}_{\text{fracture energy}} - \int_\Omega \mathbf{b}_n\cdot\mathbf{u}d\Omega - \int_{\partial\Omega_N} \mathbf{t}_n\cdot\mathbf{u}dS$$

Francfort and Marigo (1998)
Mumford and Shah (1989)

regularization ⇩ ⇧ **$\ell \to 0$ Γ−convergence**





Diffusive crack: phase-field approach

$$\min_{\mathbf{u}\in\mathcal{U}_n, d\geq d_{n-1}} \mathcal{E}_\ell(\mathbf{u},d) = \underbrace{\int_\Omega g(d)\psi(\varepsilon(\mathbf{u}))d\Omega}_{\text{elastic strain energy}} + \underbrace{\frac{G_c}{c_w}\int_\Omega\left[\frac{w(d)}{\ell} + \ell|\nabla d|^2\right]d\Omega}_{\text{fracture energy}} - \int_\Omega \mathbf{b}_n\cdot\mathbf{u}d\Omega - \int_{\partial\Omega_N} \mathbf{t}_n\cdot\mathbf{u}dS$$

Bourdin et al.(2000)
Ambrosio and Tortorelli (2000)
Braides (1998)

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The Phase-Field Method (PFM) to fracture

...with energy decomposition

$$\min_{\mathbf{u}\in\mathcal{U}_n, d\geq d_{n-1}} \mathcal{E}_\ell(\mathbf{u},d) = \int_\Omega [g(d)\psi^+(\varepsilon(\mathbf{u})) + \psi^-(\varepsilon(\mathbf{u}))]d\Omega + \frac{G_c}{c_w}\int_\Omega \left[\frac{w(d)}{\ell} + \ell|\nabla d|^2\right] d\Omega - \int_\Omega \mathbf{b}_n\cdot\mathbf{u}d\Omega - \int_{\partial\Omega_N} \mathbf{t}_n\cdot\mathbf{u}dS$$

Amor et al. (2009)
Miehe et al.(2010) Freddi and Royer-Carfagni (2010)

Phase-field evolution equation $-2l\triangle d + \frac{1}{2l}d = \frac{2(1-d)}{G_c}\mathcal{H}$ $\quad \mathcal{H}(\mathbf{x},t) := \max_{\tau\epsilon[0,t]} \Psi^+(\boldsymbol{\varepsilon}(\mathbf{x},\tau))$

$g(d)$: degradation function

ψ: energy storage function (ψ^+ refers to tension and ψ^- is compression)

G_c : fracture toughness

$w(d)$: local damage function

ℓ: crack length scale parameter

∇d: spatial gradient

$\mathbf{u}$: displacement field

c_w: normalization constant

d: fracture phase field

$\mathbf{b}_n$: body force vector $\mathbf{t}_n$: face force vector

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Project objectives

Develop a THM (Thermo-Hydro-Mechanical) crack Phase-field numerical model

- Develop a numerical model able to study the nucleation and propagation of cracks induced by **thermal effects** in multiphase heterogenous porous materials.
- Merge a **thermodynamically consistent multiphase porous media model** (and the associated finite element code Comes-Geo developed at the UNIPD) with a **crack phase-field model** (and the associated FEM code for brittle fracture developed at ETH Zurich).
- After **validation with experiments** designed from the numerical modelling, the model will be further **extended.**
- Application to study cracks in clayey materials and in heterogeneous composite materials.

Thanks for the attention

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