Università degli Studi di Padova

# Crack Propagation in Composite Aerospace Materials under Fluid-Structure Interaction Loads using a Peridynamic Approach

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Meeting - 27 October

# Motivation for Peridynamic

**Classical theory of continuum mechanics**

Some limitations for crack propagation

- It has partial derivatives with respect to spatial coordinates, which are undefined along the cracks
- To compensate this weakness, the problem is redefined along the cracks
- Not appropriate for spontaneous cracks
- Some assumptions like the time crack initiates and crack growth speed

Simulating crack propagation is a challenging task

**Finite Element Method (FEM), Extended FEM (XFEM) and Meshless Methods**

# Peridynamic

**Peridynamic** is a nonlocal continuum mechanics formulation (developed by Silling in 2000).

- Modeling problems with singularities like cracks
- Appropriate for large number of cracks
- Integration is used instead of differentiation

# Peridynamic Formulation

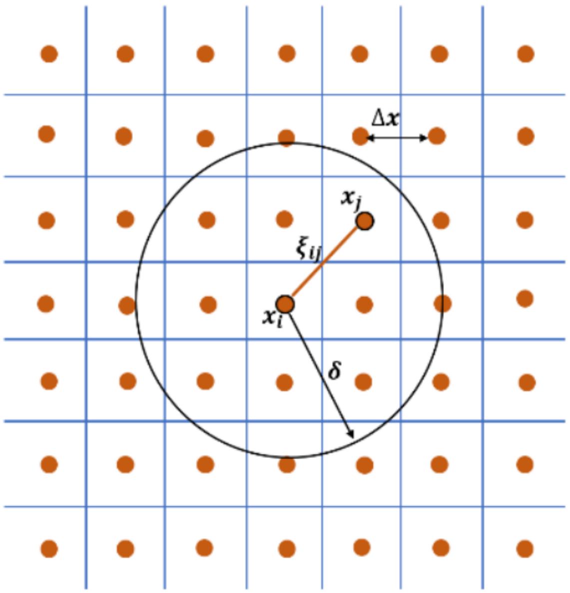
$$\rho(\boldsymbol{x})\ddot{\boldsymbol{u}}_n = \sum_j (\underline{\mathrm{T}}_n^{ij}\langle \boldsymbol{x}' - \boldsymbol{x}\rangle - \underline{\mathrm{T}}_n^{ji}\langle \boldsymbol{x} - \boldsymbol{x}'\rangle) V_{ij} + \boldsymbol{b}_n(\boldsymbol{x})$$

**Bond-based**

- The force in each bond does not depend on other bonds deformation
- Can not enforce incompressible shear deformation

**Ordinary-State-based**

- The force of each bond depends on all the bonds inside the horizon



# Motivation for Elastoplastic Approach

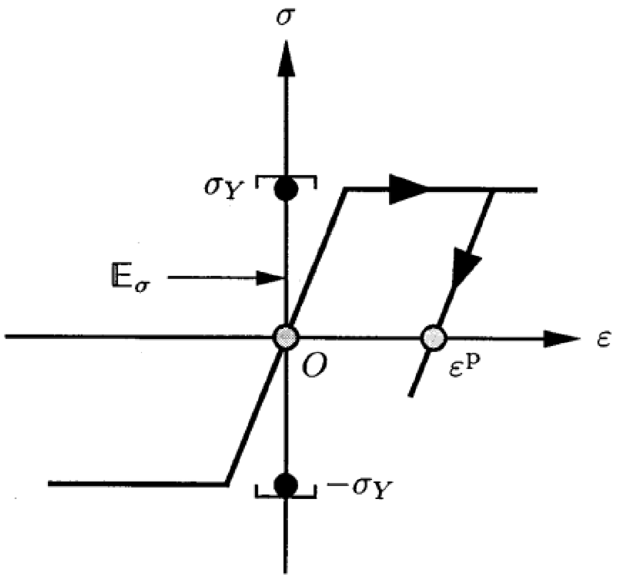


Before crack initiation, most materials experience deformation in **Plastic Domain.**

Equipping the bonds of OSB-PD with an elastoplastic constitutive law is the objective of this research.

Materials with **perfect plasticity**:

Closure of the elastic range ($E_\sigma$) remains unchanged.



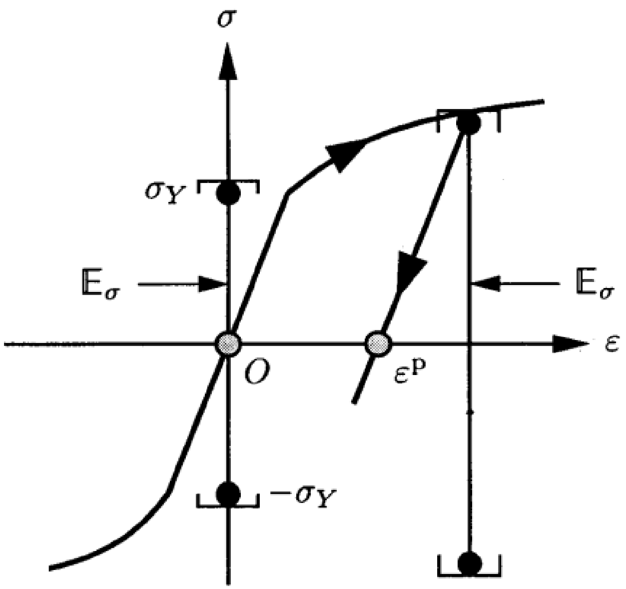
# Motivation for Elastoplastic Approach



Before crack initiation, most materials experience deformation in **Plastic Domain.**

Materials with **strain hardening**:

Closure of the elastic range ($E_\sigma$) expands with the amount of slip in plastic domain.



## Perfect Plasticity

Elastic stress-strain relationship:

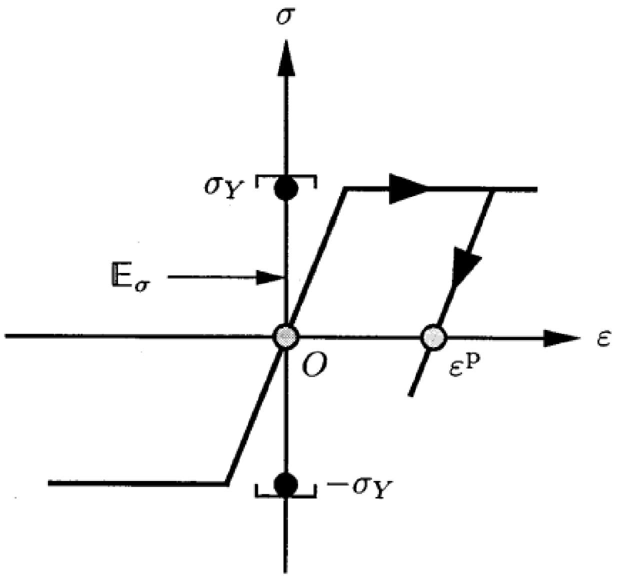
$$\sigma = E(\varepsilon - \varepsilon_p)$$

Yield condition:

$$f(\sigma) = |\sigma| - \sigma_Y \leq 0$$

Consistency condition for plastic domain:

$$\lambda \dot{f}(\sigma) = 0$$



## Plasticity with Isotropic Hardening

Elastic stress-strain relationship:

$$\sigma = E(\varepsilon - \varepsilon_p)$$

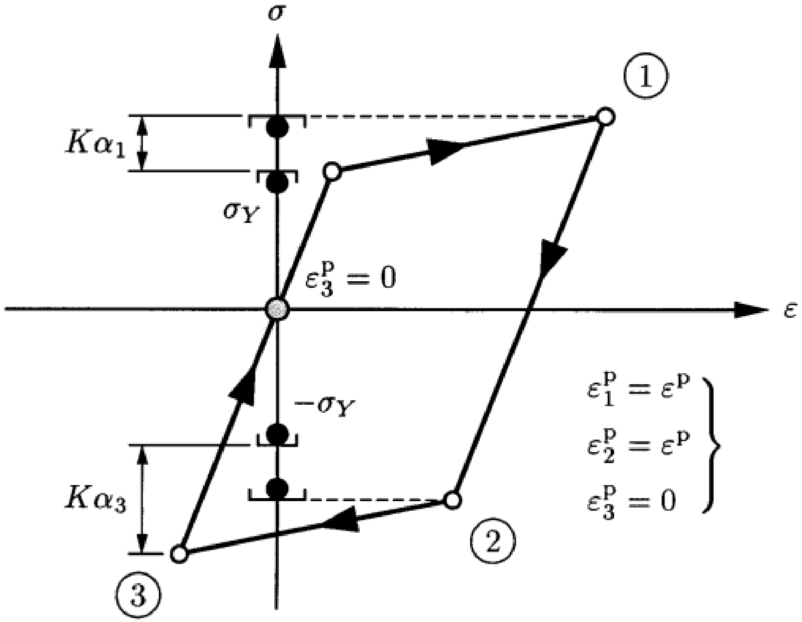
Yield condition:

$$f(\sigma,\alpha) = |\sigma| - (\sigma_Y + K\alpha) \le 0$$

Consistency condition for plastic domain:

$$\lambda \dot{f}(\sigma,\alpha) = 0$$

The center of the yield surface is not changed.



## Plasticity with Kinematic Hardening

Elastic stress-strain relationship

$$\sigma = E(\varepsilon - \varepsilon_p)$$

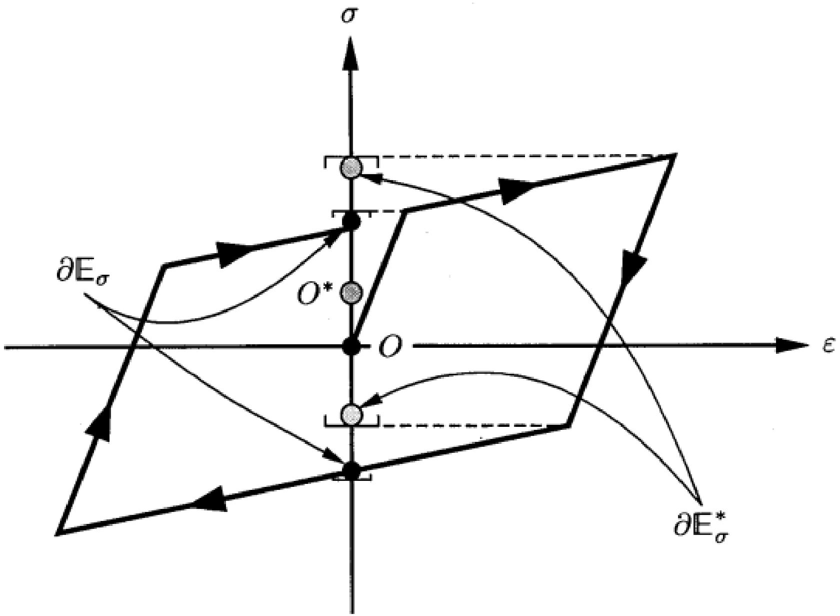
Yield condition

$$f(\sigma, q) = |\sigma - q| - \sigma_Y \leq 0$$

Consistency condition for plastic domain:

$$\lambda \dot{f}(\sigma, q) = 0$$

Center of the yield surface moves in the direction of the plastic flow (Bauschinger effect)



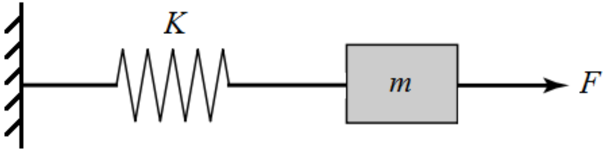
# Elastoplastic Analysis of SDOF (Perfect Plasticity)

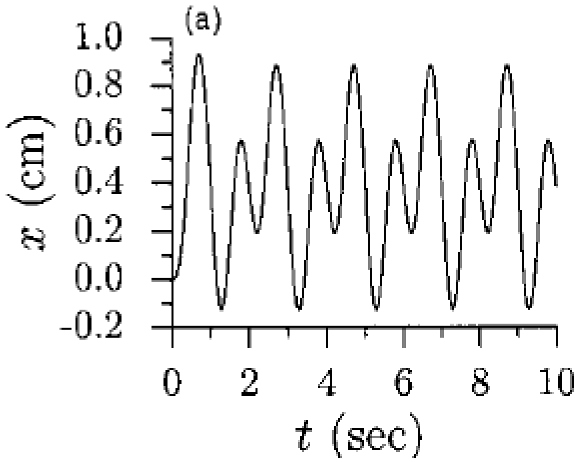
$$K = 1000\pi^2 \; N/m \qquad t = 10s$$

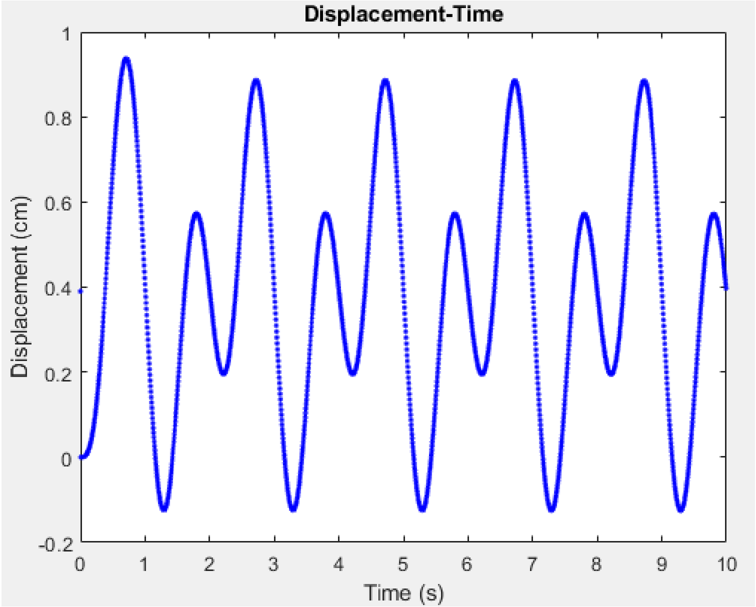
$$m = 1000\,kg \qquad \Delta t = 5\,ms$$

$$F_Y = 50\,N \qquad N = 2000$$

$$F = 100\sin(2\pi t) \;\; (N)$$



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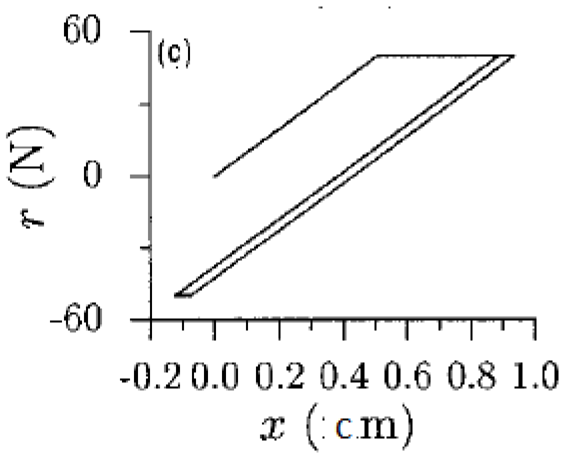
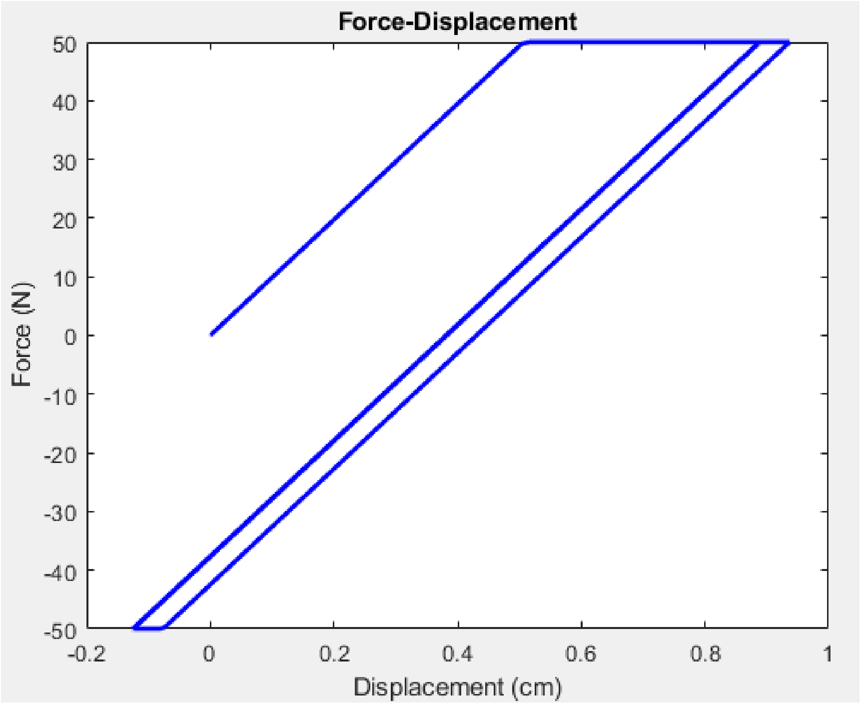


# Elastoplastic Analysis of SDOF (with Kinematic Hardening)



$K = 350000\, kN/m$ $\qquad t = 2s$

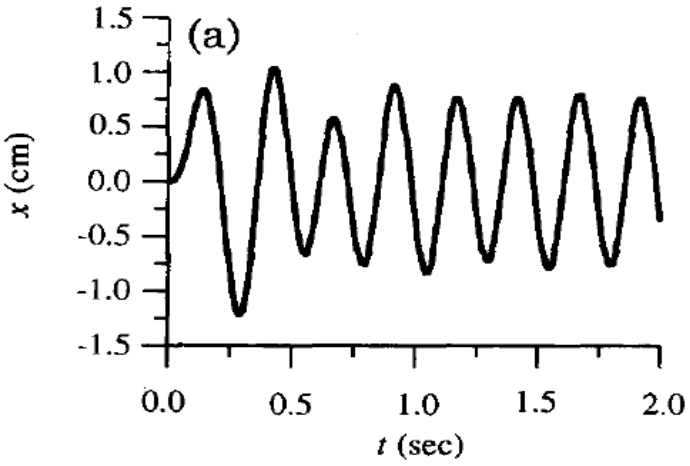
$H = 100000\, kN/m$ $\qquad \Delta t = 1ms$

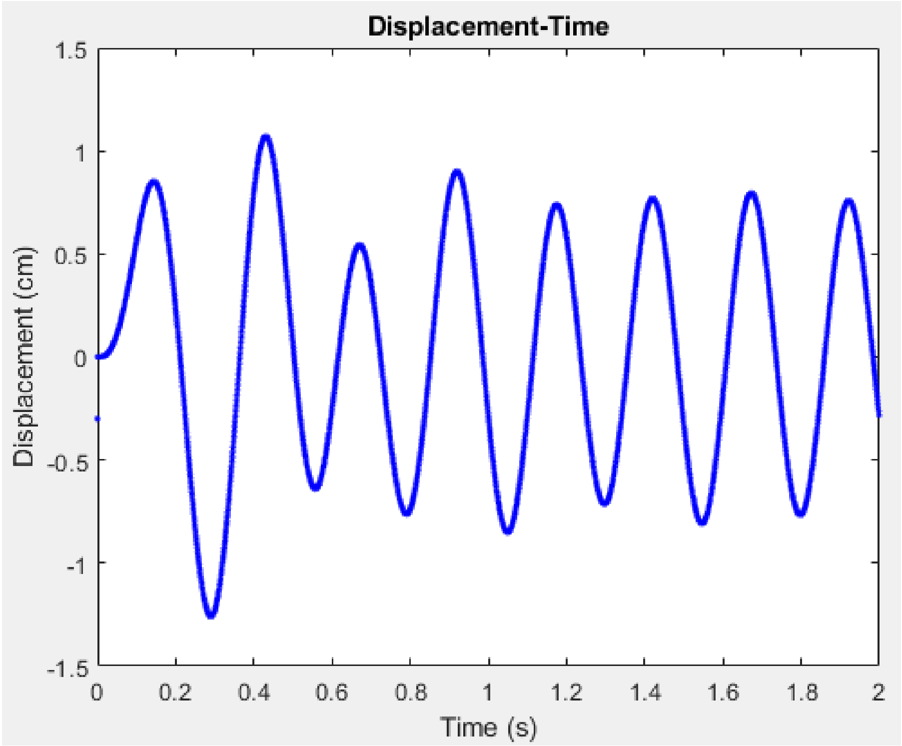
$c = 350\, kN.s/m$ $\qquad N = 2000$

$m = 350\, tonne$

$F_Y = 400\, kN$

$F = 1000 \sin(8\pi t)\ \ (N)$



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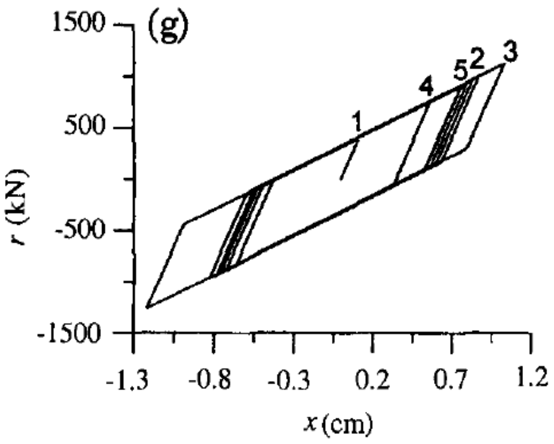
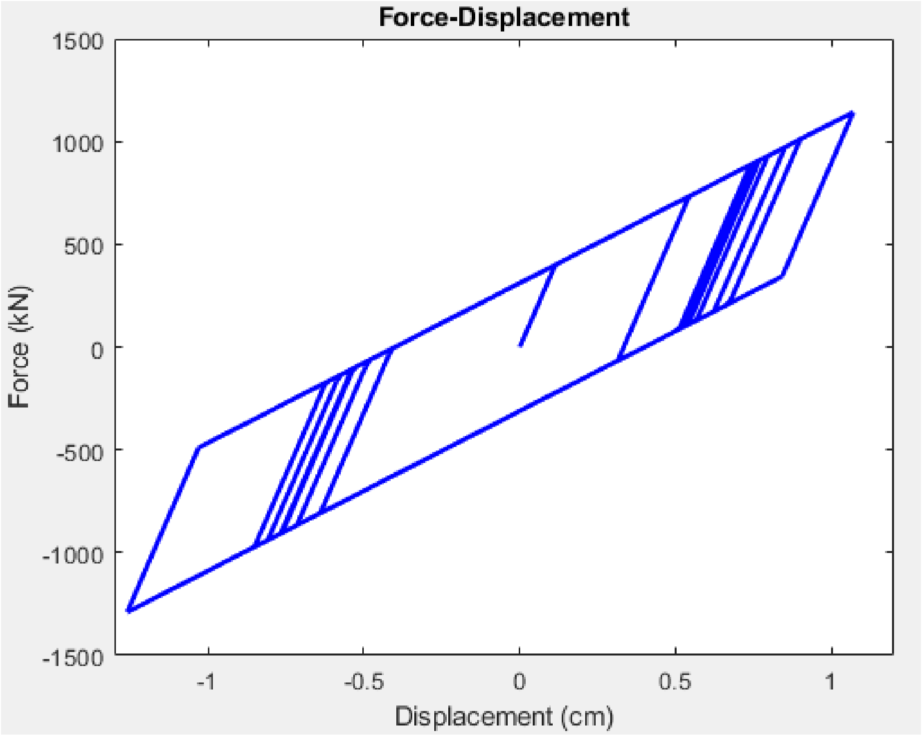


# Elastoplastic Analysis of SDOF (with Isotropic Hardening)



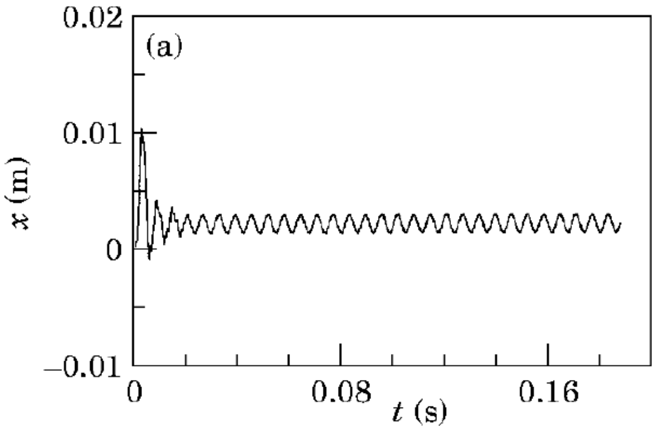
$$K = 54.9 \times 10^{6}\ N/m \qquad t = 0.2s$$

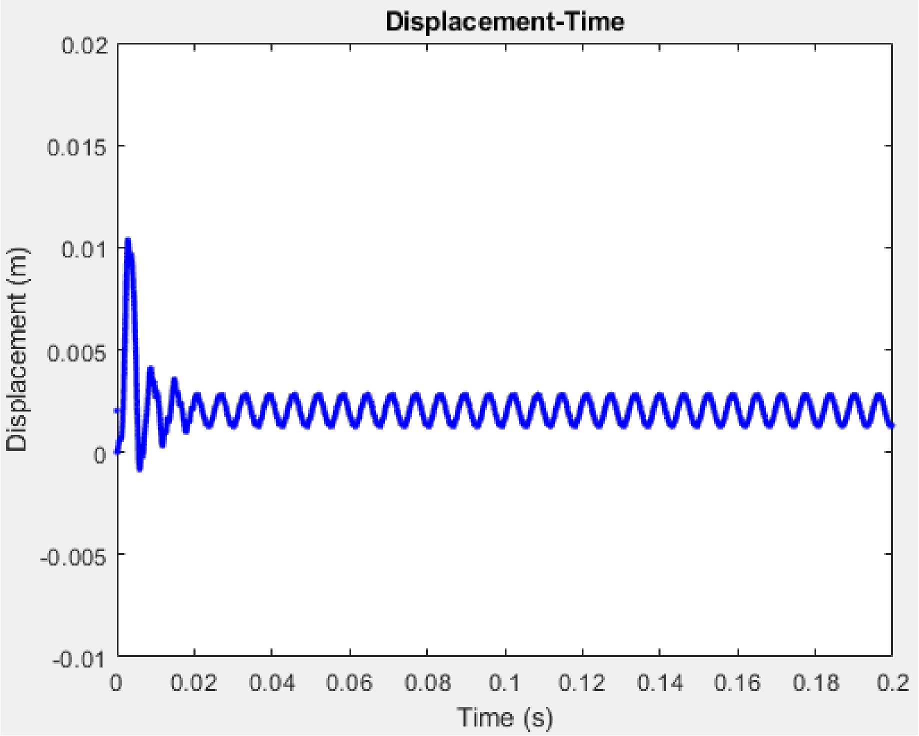
$$Ki = 345.6 \times 10^{3}\ N/m \qquad \Delta t = 16\ \mu s$$

$$m = 1\ kg \qquad N = 12500$$

$$F_{Y} = 31.4 \times 10^{3}\ N$$

$$F = 41400 \sin(1000t)\ (N)$$



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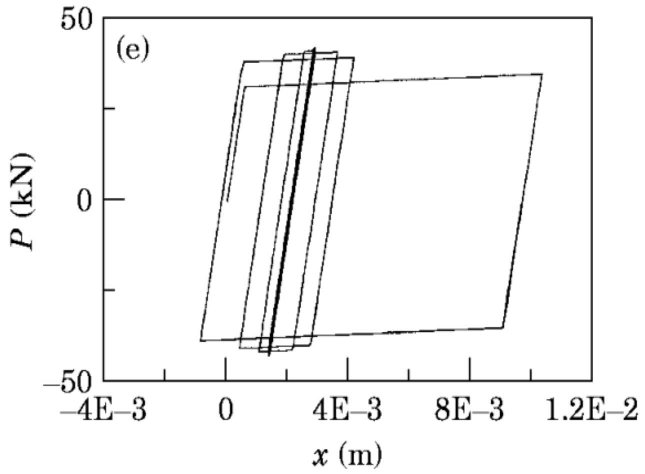
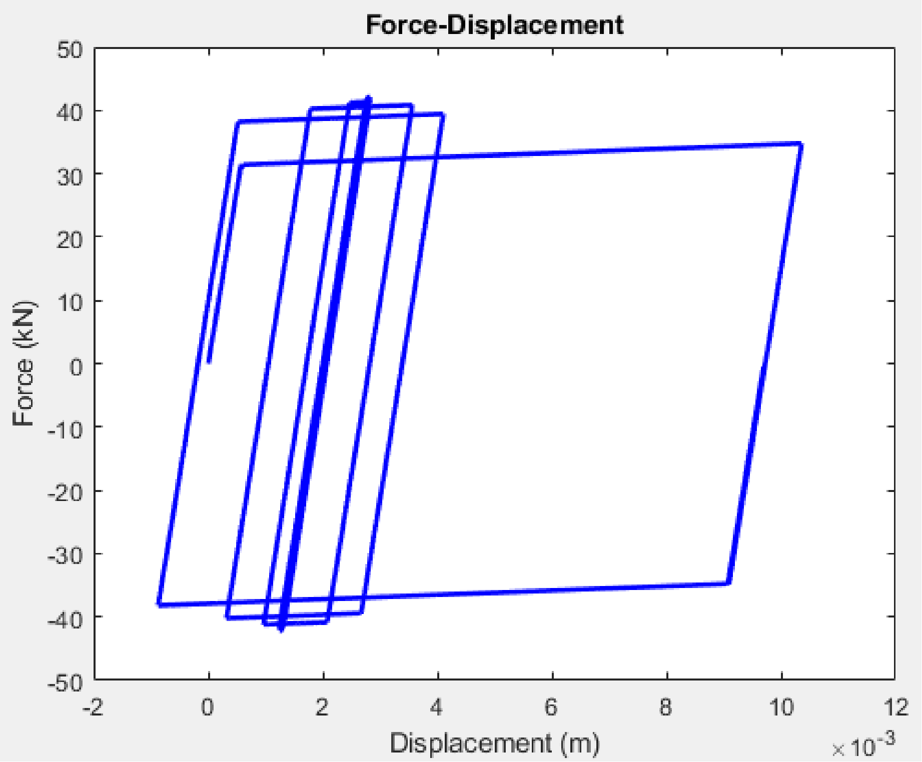


# State-based Elastoplastic Analysis

In J2 plasticity, the von-Mises yield criterion is used based on maximum deviatoric strain energy. for 2D case:

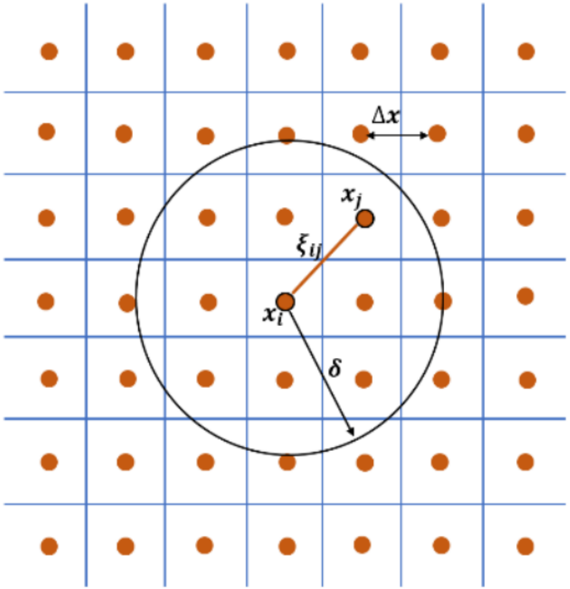
$$f\,(\underline{t}^{d}_{trial}) = \frac{\left\|\underline{t}^{d}_{trial}\right\|^2}{2} + 4\frac{\left(\underline{t}^{d}_{trial} \bullet \underline{x}\right)^2}{\pi\delta^4 l_z} - \psi_0$$

$$if\ \ f\,(\underline{t}^{d}_{trial}) \le 0 \ \Rightarrow\ elastic\ step\ \ (\Delta\lambda = 0) \Rightarrow \begin{cases} t^{d}_{n} = t^{d}_{trial} \\ \underline{e}_n = \underline{e}_{trial} \end{cases}$$

$$if\ \ f\,(\underline{t}^{d}_{trial}) > 0 \ \Rightarrow\ plastic\ step\ (\Delta\lambda \ne 0) \Rightarrow \begin{cases} f\,(\underline{t}^{d}) = 0 \\ \lambda \dot{f}\,(\underline{t}^{d}) = 0 \end{cases}$$

$$\underline{e}^{dp}_{n} = \underline{e}^{dp}_{n-1} + \Delta\lambda \underline{t}^{d}_{n}$$

Dynamic relaxation method is used to calculate next step trial bond extension and bond force.



# Conclusion

- A thorough bibliographic research in Peridynamics and elastoplastic analysis has been done.
- State-based **Peridynamics** will be used to study crack propagation in **elastoplastic materials**.
- Ordinary state-based model will be used to consider the effects of shear deformation in plastic domain.
- The purpose is to equip the bonds of **ordinary state-based Peridynamics** with an elastoplastic constitutive law.
- Dynamic relaxation method will be used to obtain steady-state solutions of nonlinear peridynamic equations.
- The elastoplastic Peridynamic approach will be used for analysis of the problems with fracture under fluid-structure interaction loads.

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# Thanks for the attention