



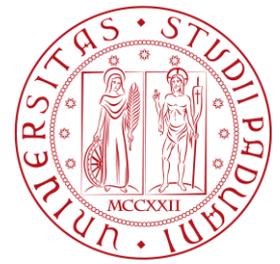
# Investigation on fatigue damage laws for the study of delamination in composite materials

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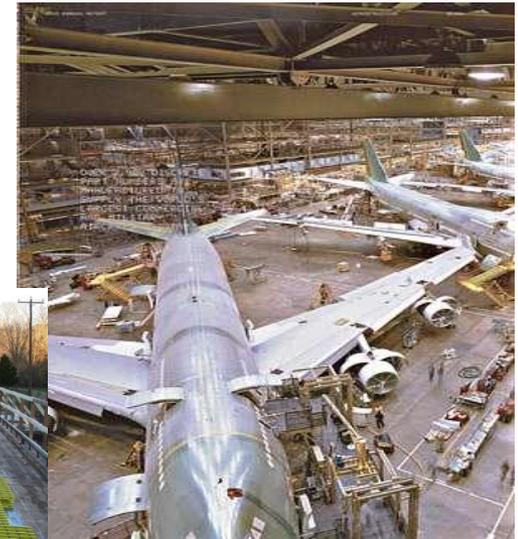
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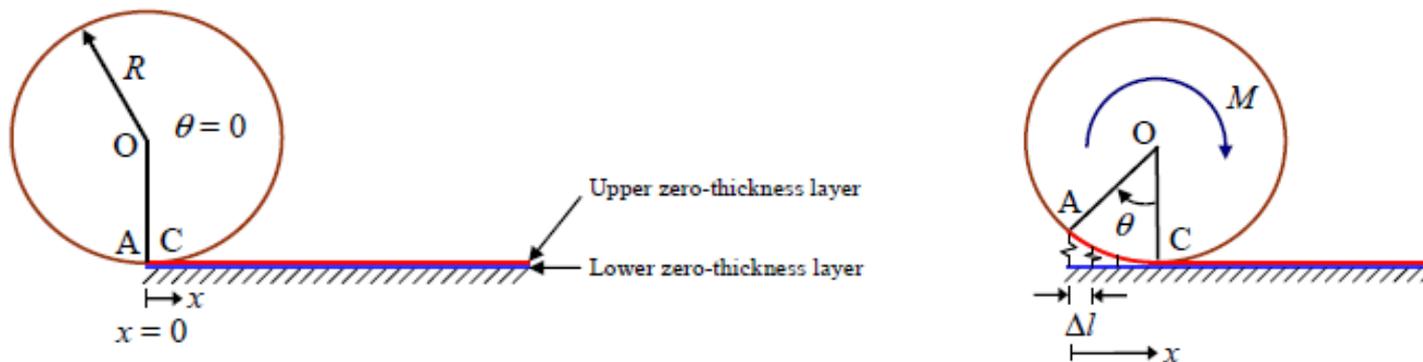
# Means of composite materials

- Light weight structures with high resistance and stiffness
- Ocean, Maritime in coastal engineering
- Aerospace engineering
- Industrial engineering and transportation industry



# Cylinder model

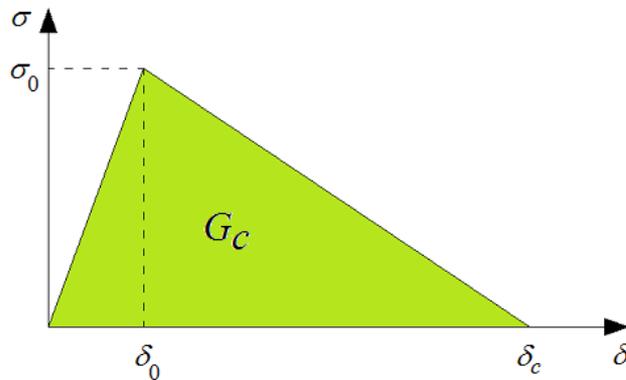
- Consider a cylinder of radius  $R$  which, under static loading, rolls in a clockwise direction on a horizontal surface.
- The lower layer is fixed. The upper layer adheres to the surface of the cylinder at the contact point,  $C$  and remains adhered.
- As the cylinder rotates and the contact point advances as the cylinder rolls along the horizontal surface the angle swept through by  $OA$  is the rotation,  $\theta$ , of the cylinder.



## Interface constitutive laws

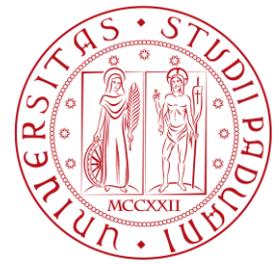
### 1- Bilinear interface constitutive law

$K$  is the stiffness:  $K = \sigma_0 / \delta_0$



$$\begin{cases} \sigma = K\delta & 0 \leq \delta \leq \delta_0 \\ \sigma = \sigma_0 \left( \frac{\delta_c - \delta}{\delta_c - \delta_0} \right) & \delta_0 < \delta < \delta_c \\ \sigma = 0 & \delta_c < \delta \end{cases}$$

The critical energy release rate:  $\Rightarrow G_c = \frac{\delta_c \sigma_0}{2}$

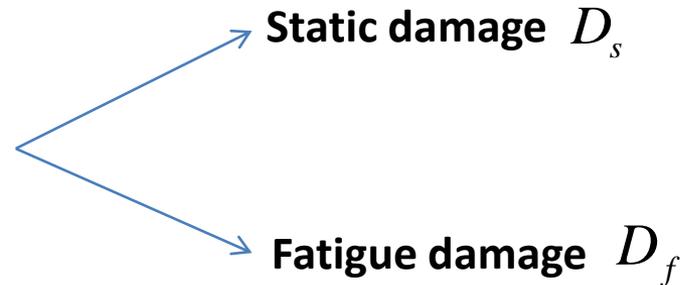


## Interface constitutive laws in terms of a damage variable

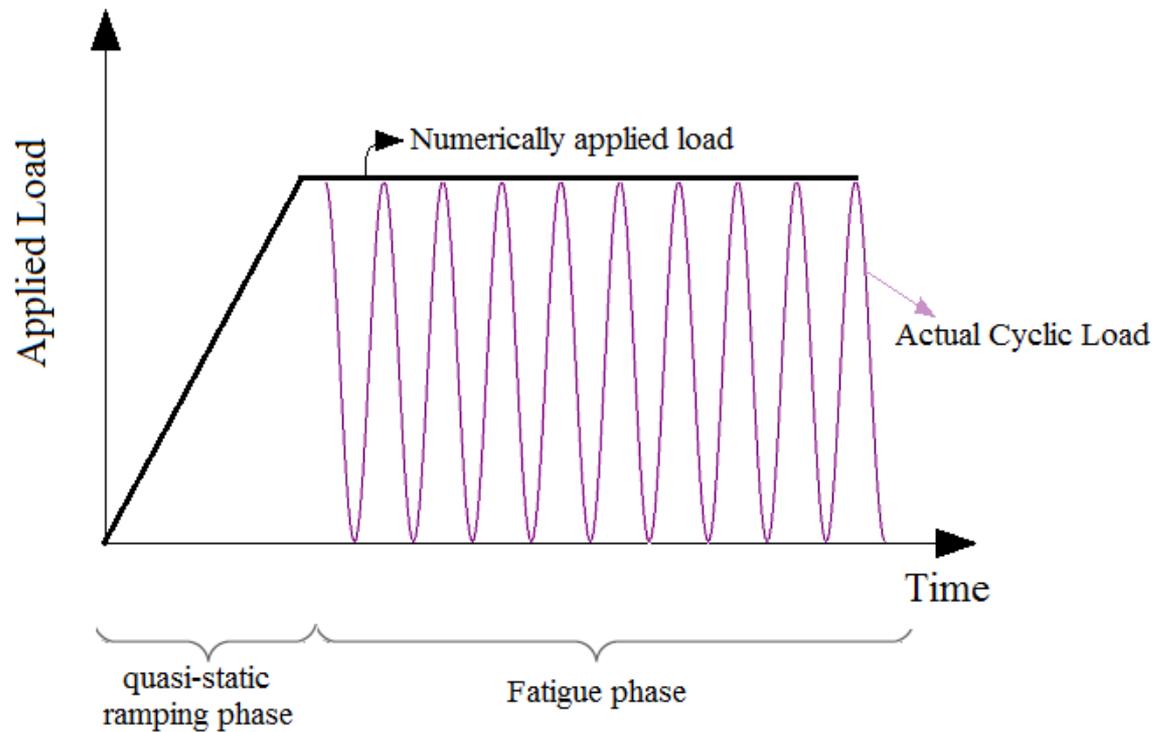
Damage is defined so that for the undamaged state it is equal to zero and for the failure state it is equal to unity

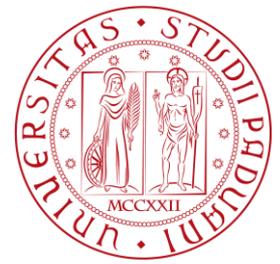
$$\begin{cases} D = 0 & \text{undamaged} \\ 0 < D < 1 & \text{damaged} \\ D = 1 & \text{failed} \end{cases}$$

Two components of damage are considered



## Envelope of the assumed cyclic load for a large number of cycles





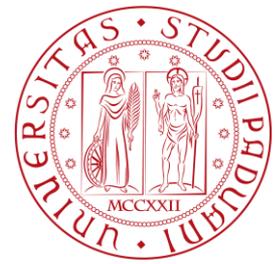
## Stiffness degradation based static damage

In this approach the static damage,  $D_s$ , can be considered a measure of the degradation of the initial stiffness

$$\sigma = (1 - D_s) K \delta$$

$D_s$  can be derived

$$D_s = \begin{cases} 0 & 0 \leq \delta \leq \delta_0 \\ \left( \frac{\delta - \delta_0}{\delta} \right) \left( \frac{\delta_c}{\delta_c - \delta_0} \right) & \delta_0 < \delta < \delta_c \\ 1 & \delta \geq \delta_c \end{cases}$$



## Static and Fatigue degradation strategies

□ Rate of change of the static damage


$$\frac{\partial D_s}{\partial t} = \frac{\delta_0 \delta_c}{\delta_c - \delta_0} \frac{\dot{\delta}}{\delta^2} \quad \delta_0 < \delta < \delta_c$$

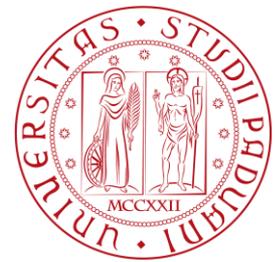
$$D_{sN+\Delta N} - D_{sN} = \frac{\delta_0 \delta_c}{\delta_c - \delta_0} \left( \frac{1}{\delta_N} - \frac{1}{\delta_{N+\Delta N}} \right)$$

□ **Fatigue** damage Strategies


$$(1) D_{fN+\Delta N} - D_{fN} = \Delta N \frac{C}{1+\beta} e^{\lambda D_\mu} \left( \frac{\delta_\mu}{\delta_c} \right)^{1+\beta}$$


$$(2) D_{fN+\Delta N} - D_{fN} = \frac{\delta_0 \delta_{cN+\Delta N}}{\delta_{cN+\Delta N} - \delta_0} \left( \frac{1}{\delta_0} - \frac{1}{\delta_{N+\Delta N}} \right)$$

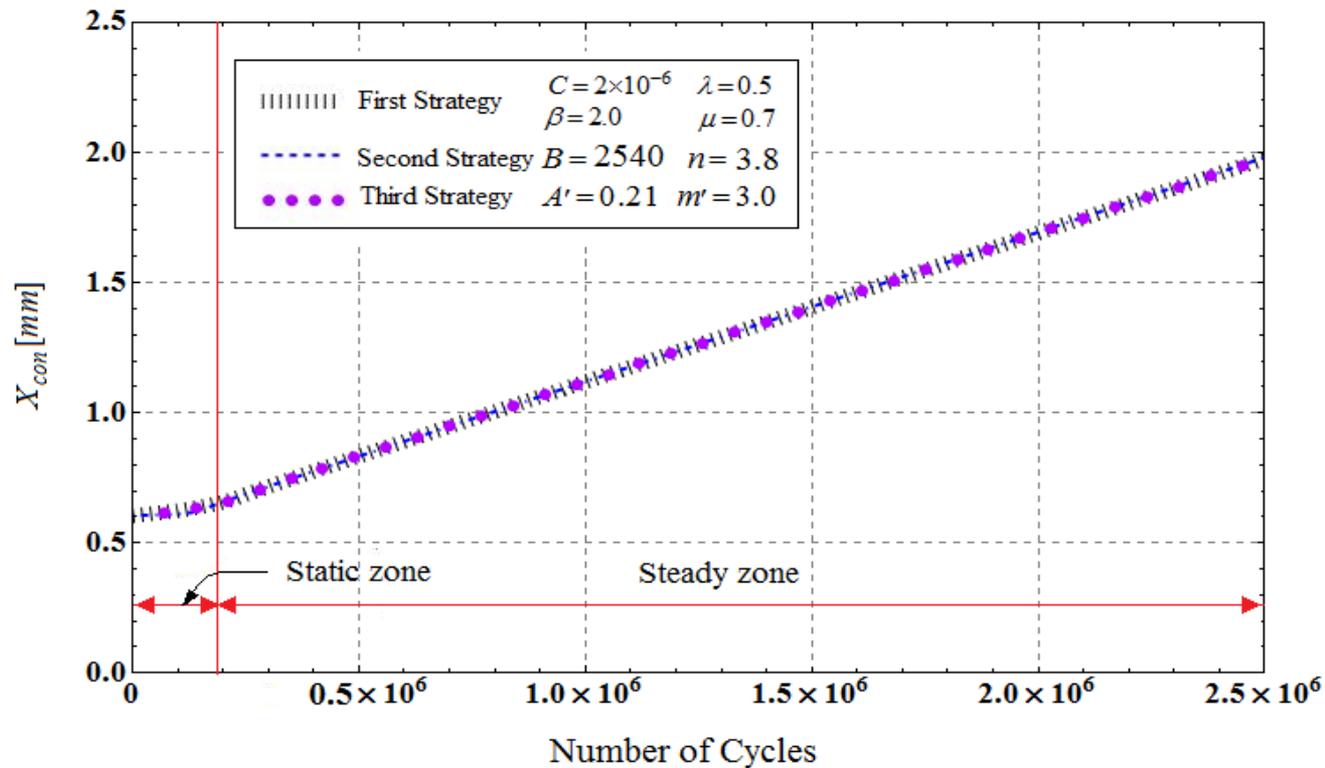

$$(3) D_{fN+\Delta N} - D_{fN} = A' \cdot \Delta N (\delta_{N+\Delta N})^{m'}$$

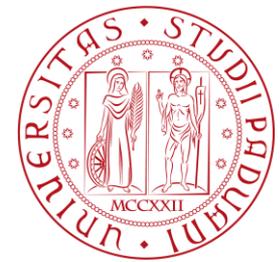


## Examples of fatigue behavior simulation

$$R = 100\text{mm} \quad G_c = 0.26\text{N/mm} \quad \delta_c = 0.017333\text{mm}$$

$$\Delta l = 0.005\text{mm} \quad \Delta N = 100 \quad M_a = 0.2M_c$$





## Total amount of time

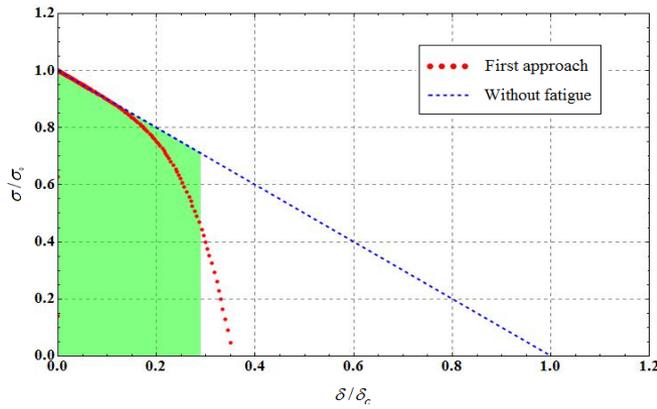
This Problem has been solved by an Intel(R) Core(TM) i7-2600K CPU @ 3.40GHz machine and the computational time reported are based on the CPUsec of this system

**First Approach** : 67.92 CPUs

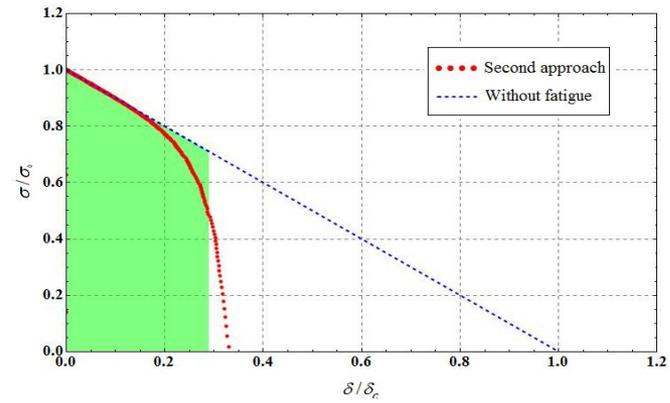
**Second Approach**: 59.45 CPUs

**Third Approach**: 57.00 CPUs

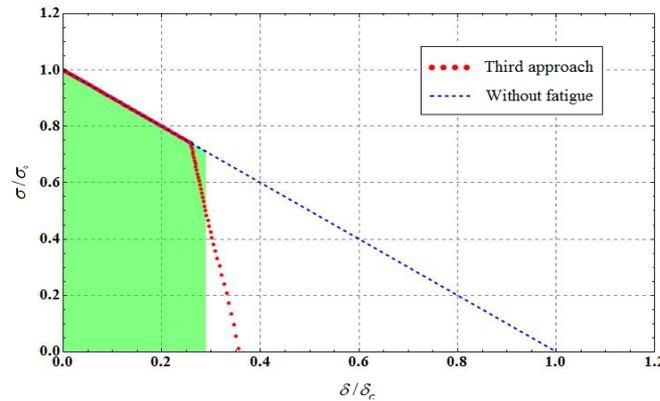
## Single spring behavior under static and fatigue loading using three proposed laws



**First approach**

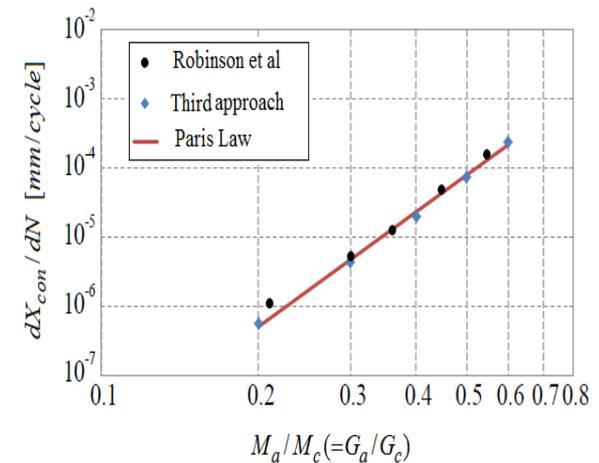
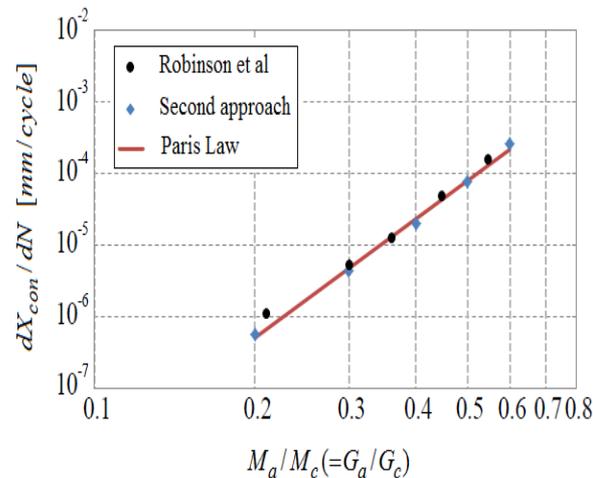
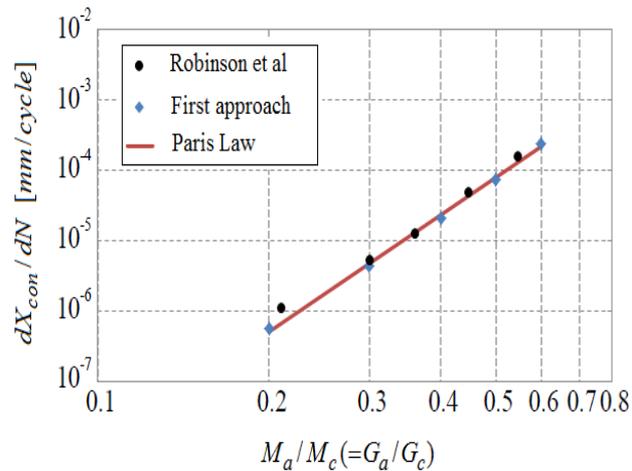


**Second approach**



**Third approach**

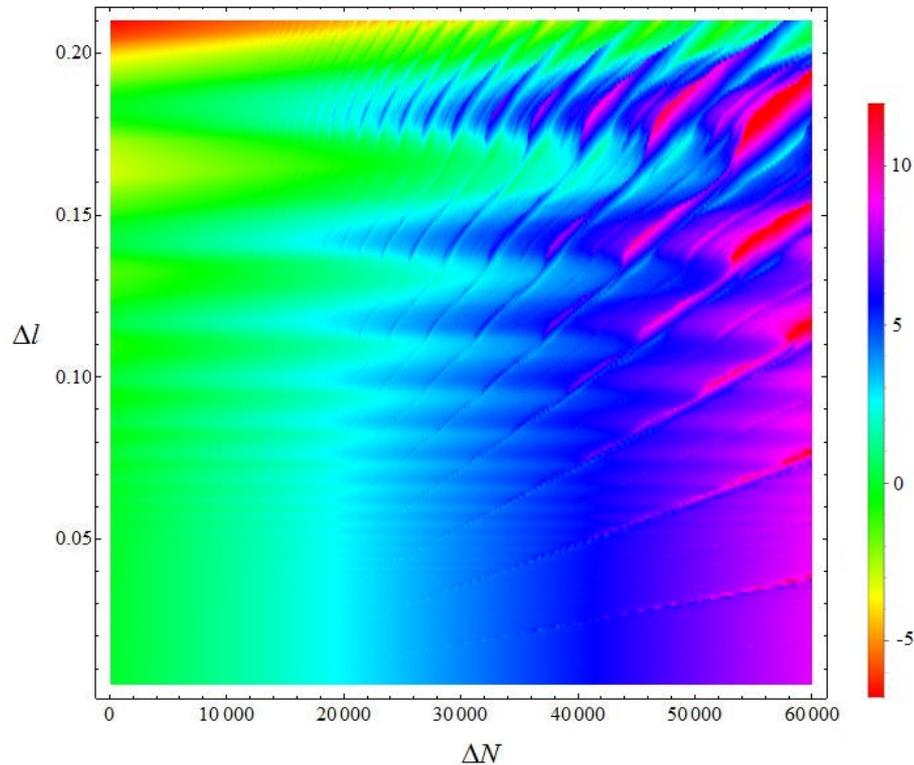
## All proposed fatigue approaches are capable to describe a behavior similar to the Paris Law



If the slope  $dx_c/dN$  for different values of the applied moment is plotted in a log-log scale against the ratio of the applied moment vs. the critical moment, a Paris plot is obtained

- ❑ ‘Exact slope’ ( $dx_c/dN$ ) computed numerically using a small value of  $\Delta l$  ( $=0.005$ ) and  $\Delta N$  ( $=100$ ).
- ❑ The value of the ‘exact slope’ is approximately the same for all Three formulations.

### Error plots for first fatigue Law

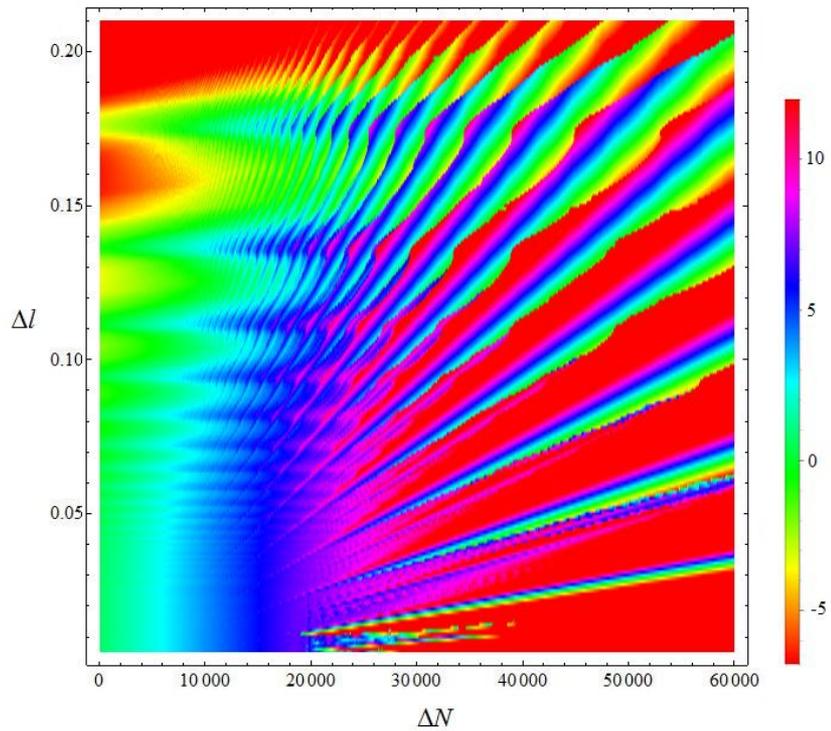


- ❑ The range of  $\Delta l$  values has been divided in 200 uniform subintervals and the range of  $\Delta N$  values in 600 uniform subintervals so that 120000 different parameter combinations have been considered for each case.

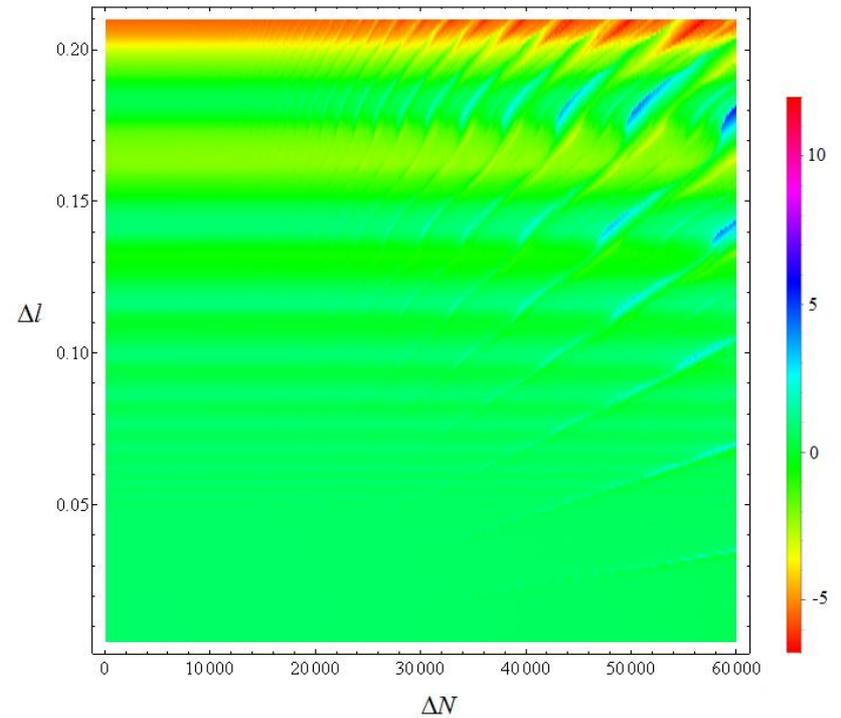
$$0.005 < \Delta l < 0.2$$

$$100 < \Delta N < 60000$$

**Error plots for second fatigue Law**

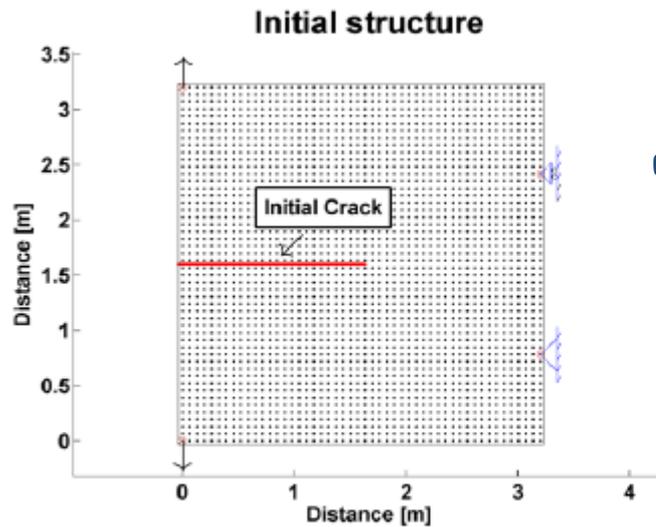


**Error plots for third fatigue Law**

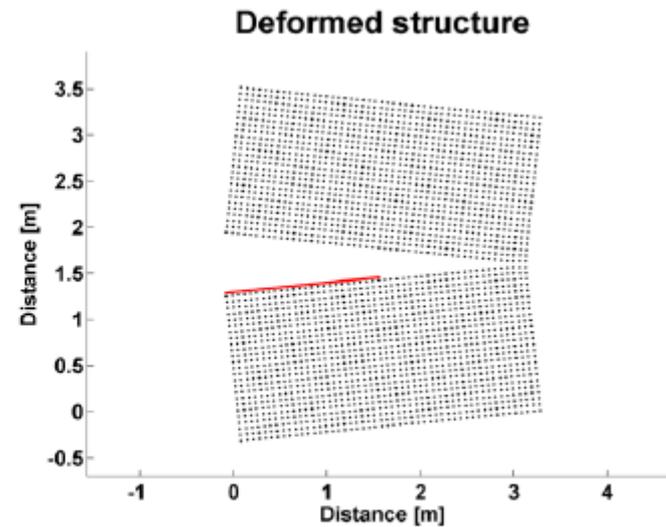


## Future works

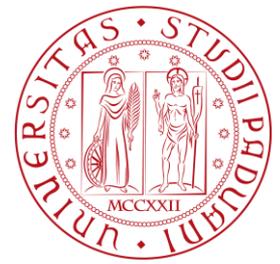
- How to implement the proposed laws in real structures using peridynamic approach
- Investigating crack propagation due to the High Cycle Fatigue laws
- Solving the problem Using Newton-Raphson scheme or Dynamic Relaxation



(a)



(b)



## Conclusions

- ❑ A **simple cylinder model** has been introduced to compare in a systematic, fast and rational manner the behavior of interface elements used to **simulate fatigue crack** propagation in composite materials.
- ❑ Three **fatigue degradation strategies have been coupled to the various formulations** and these have been implemented in the simple model to demonstrate its effectiveness.
- ❑ The results show that the cylinder model can be used to readily assess the effectiveness of fatigue-driven delamination growth strategies. For the limited number of formulations considered, the results of the model suggest that the bilinear constitutive model with the stiffness degradation damage formulation provides the best performance. The simple cylinder model is currently being used to **explore the effectiveness of other fatigue-driven delamination growth strategies**.

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**Thanks for Your  
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