Università degli Studi di Padova

Dipartimento di Ingegneria Industriale

On the Coupling of Peridynamics with the classical theory of continuum mechanics in a meshless framework

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Admission to the final exam, cycle 30th

Padua - Italy

Octobre, 2017

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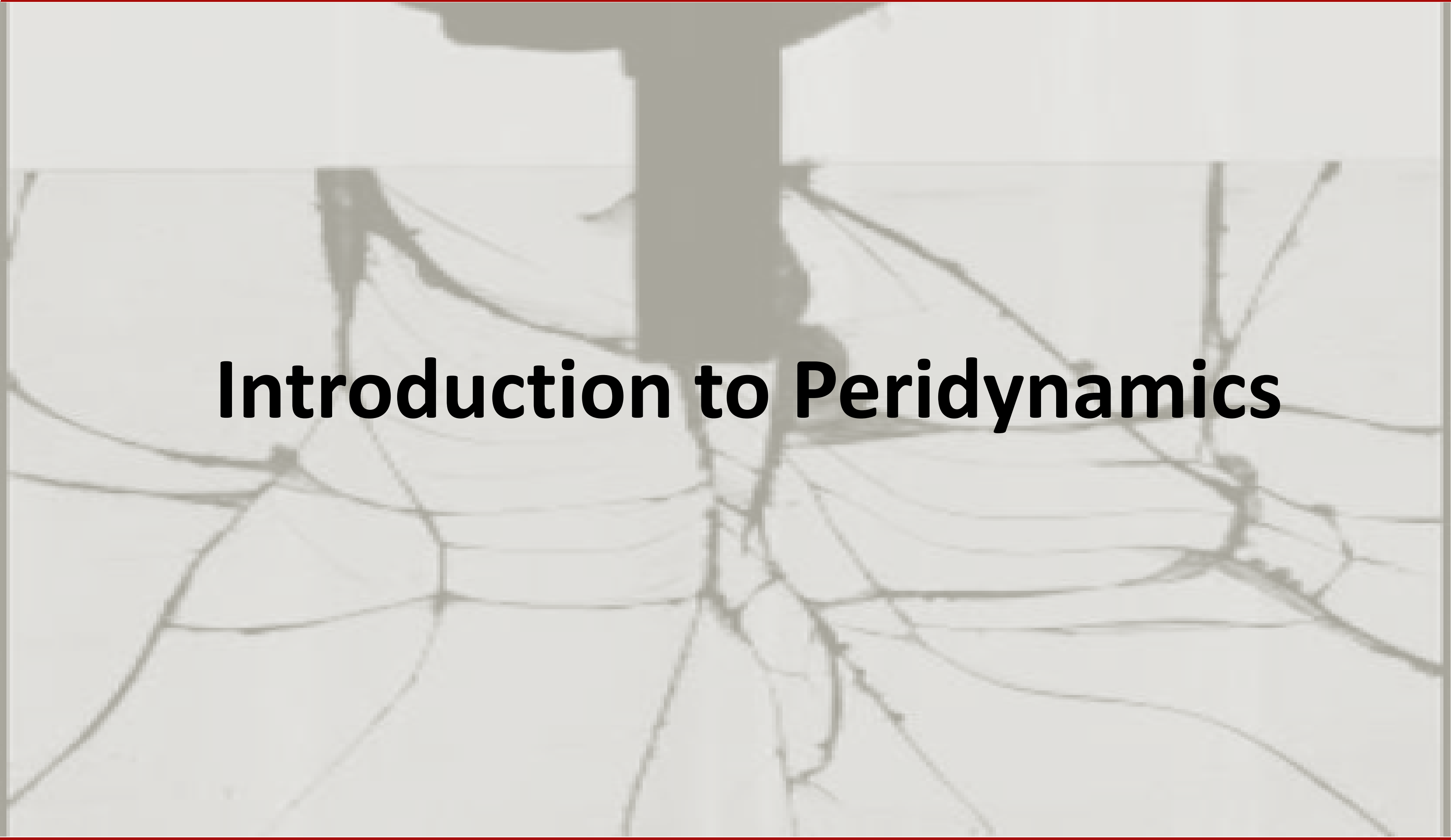


Introduction to Peridynamics

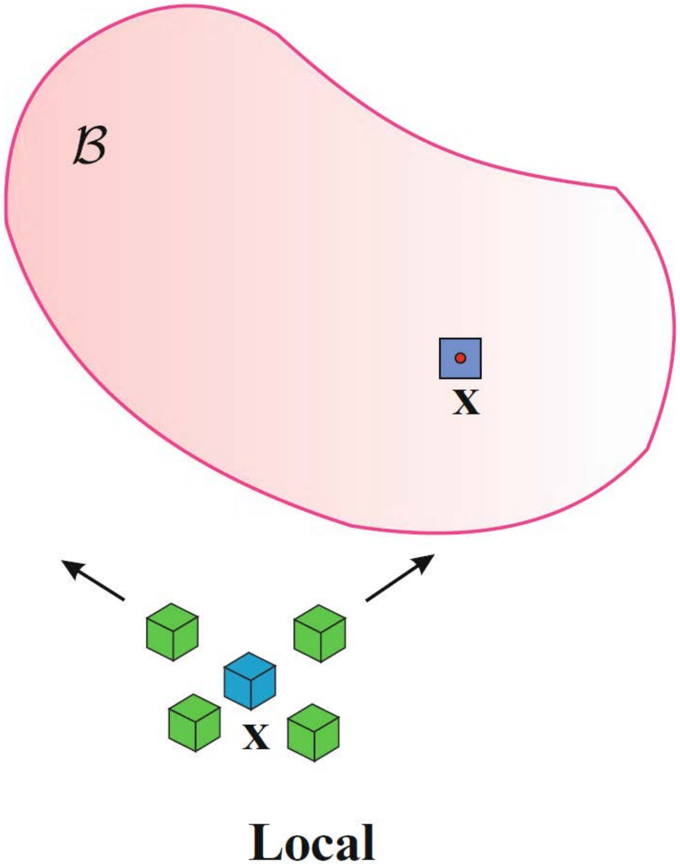
Arman Shojaei, Mirco Zaccariotto and Ugo Galvanetto, University of Padova, Padova, Italy

Introduction to Peridynamics: Fundamentals

The classical theory of solid mechanics:

- It is a local theory.
- It assumes as body deforms it remains continuous.
- The stress state depends on the deformation at point.
- The equations of motion contain partial derivatives.

$$\rho \ddot{\mathbf{u}} = div\mathbf{T} + \mathbf{b}$$



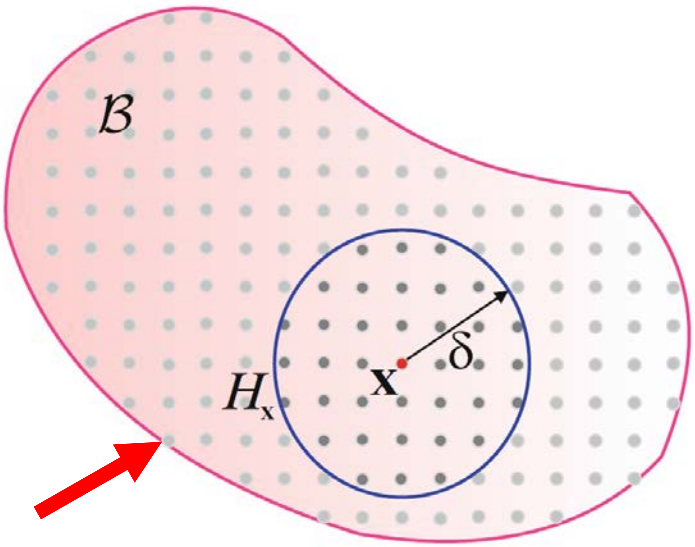
❖ The validity breaks down when singularities such as cracks come into the picture.

Introduction to Peridynamics: Fundamentals

The Peridynamic theory:

- It is a nonlocal theory.
- No a-priory knowledge about the crack initiation is required.
- Crack is free to arise and grow in every part of the structure.

$$\rho \ddot{\mathbf{u}}(\mathbf{x}_i, t) = \int_{H_x} \mathbf{f}(\mathbf{u}(\mathbf{x}', t) - \mathbf{u}(\mathbf{x}, t), \mathbf{x}' - \mathbf{x}) dV + \mathbf{b}(\mathbf{x}, t)$$

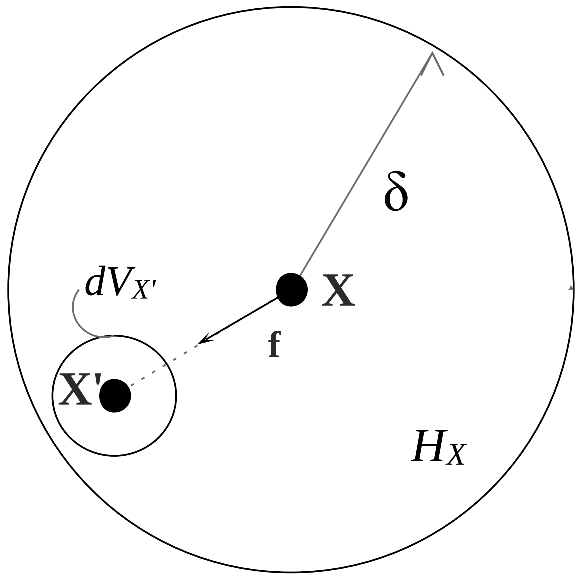


Peridynamics

❖ The theory is formulated free of partial derivatives and hence the equations of motion hold every where regardless of the discontinuities.

Introduction to Peridynamics: Fundamentals

The pairwise force function expresses the vector force between **x** and **x**′:



$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = f\,\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\left\|\boldsymbol{\xi}+\boldsymbol{\eta}\right\|}$$

Definitions:

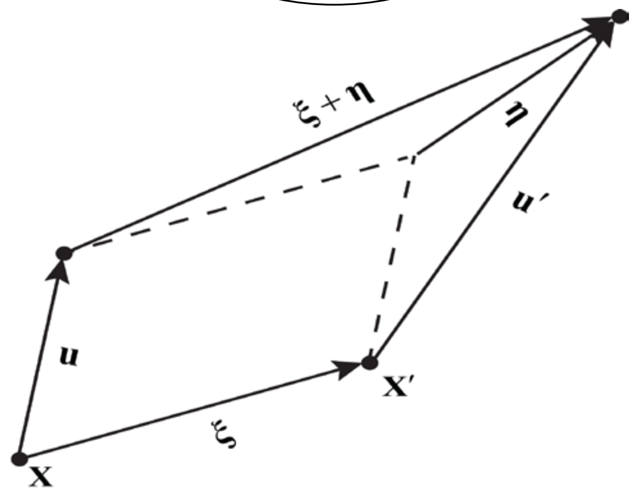
$\mathbf{u}$: displacement vector field

ρ: mass density

$\mathbf{b}$: body force density

$\boldsymbol{\xi} = \mathbf{x}' - \mathbf{x}$: initial relative position

$\boldsymbol{\eta} = \mathbf{u}' - \mathbf{u}$: relative displacement



Introduction to Peridynamics: Fundamentals

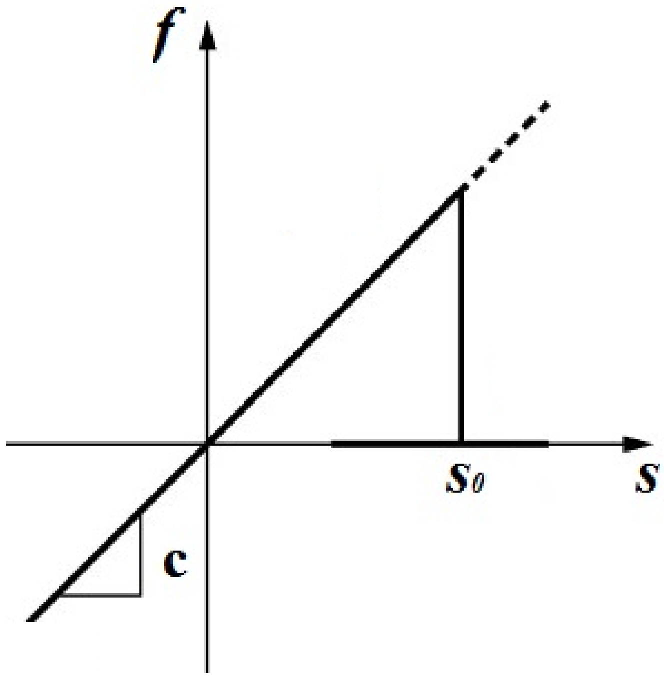
The pairwise force function is defined for a ***linear elastic material***, by means of the following expression (considering a planer state):

$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = cs\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\|\boldsymbol{\xi}+\boldsymbol{\eta}\|}$$

$$c \propto \frac{E}{\delta^3} \quad \rightarrow \quad \text{bond stiffness}$$

$$s = \frac{\|\boldsymbol{\xi}+\boldsymbol{\eta}\| - \|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|} \quad \rightarrow \quad \text{stretch}$$

$$s_0 \propto \sqrt{G_0/E\delta} \quad \rightarrow \quad \text{failure bond stretch}$$

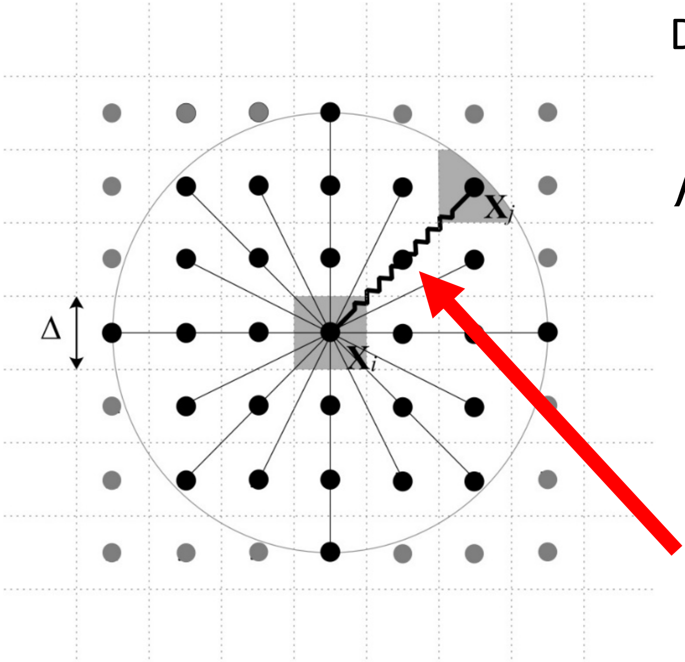


$E \rightarrow$ Module of elasticity

$G_0 \rightarrow$ Fracture Energy

Introduction to Peridynamics: Discretization

In the numerical implementation of the Peridynamic approach, the body is discretized into grid points called **nodes**.



Discretized equation of motion :

$$\rho \ddot{\mathbf{u}}_i^n = \sum_j \mathbf{f}(\mathbf{u}_j^n - \mathbf{u}_i^n, \mathbf{x}_j - \mathbf{x}_i)\beta_j V_j + \mathbf{b}_i^n$$

Small deformation assumption:

$$\mathbf{k}_{ij} = \frac{c}{|\boldsymbol{\xi}|}\beta_j V_i V_j \begin{bmatrix} \xi_1^2 & \xi_1\xi_2 & -\xi_1^2 & -\xi_1\xi_2 \\ & \xi_2^2 & -\xi_1\xi_2 & -\xi_2^2 \\ & & \xi_1^2 & \xi_1\xi_2 \\ & & & \xi_2^2 \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{ij}^{11} & \mathbf{k}_{ij}^{12} \\ \mathbf{k}_{ij}^{21} & \mathbf{k}_{ij}^{22} \end{bmatrix}$$

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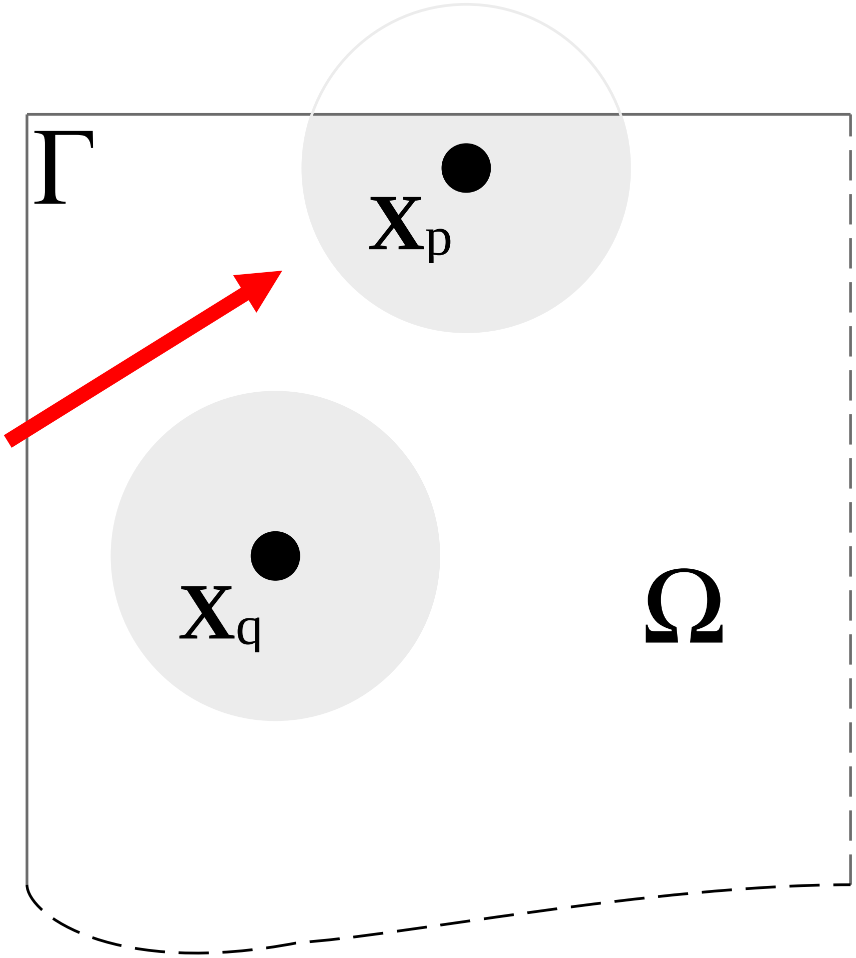
Introduction to Peridynamics: Some issues

- It requires nonlocal integrations and multiple interaction of a material point with multiple neighbors which contributes to a significant computational cost in comparison to conventional methods based on the local theory
- It suffers from the ***surface effect*** problem
- It requires a dense discretization of the solution domain as it inherits a nonlocal dispersion effect



The coupling technique

The coupling technique: the main idea

The main idea is to **couple** a meshless Peridynamic method with a meshless method based on the classical continuum theory to **restrict** the application of the nonlocal theory to the **necessary parts** (cracked parts) of the solution domain.

Concerns:

- ✓ To preserve the meshless features of the method (No introduction of interface elements).
- ✓ To be simple and free of any blending function and numerical artifacts.
- ✓ To to be free of surface effect close to the interface region
- ✓ To be free from ghost forces in in interface region
- ✓ Coupling with a strong form meshless method with cheap computational cost

FPM & **MLEBF**

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The coupling techn

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Coupling of 2D discretized Peridynamics with a meshless method based on classical elasticity using switching of nodal behaviour

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Mirco Zaccariotto and Ugo Galvanetto
Department of Industrial Engineering, University of Padua, Padova, Italy

Abstract

Purpose – The paper aims to use a switching technique which allows to couple a nonlocal bond-based Peridynamic approach to the Meshless Local Exponential Basis Functions (MLEBF) method, based on classical continuum mechanics, to solve planar problems.

Design/methodology/approach – The coupling has been achieved in a completely meshless scheme. The domain is divided in three zones: one in which only Peridynamics is applied, one in which only the meshless method is applied and a transition zone where a gradual transition between the two approaches takes place.

Findings – The new coupling technique generates overall grids that are not affected by ghost forces. Moreover, the use of the meshless approach can be limited to a narrow boundary region of the domain, and in this way, it can be used to remove the "surface effect" from the Peridynamic solution applied to all internal points.

Originality/value – The current study paves the road for future studies on dynamic and static crack propagation problems.

Keywords Meshless, Coupling, Nonlocal theory, Peridynamics, Surface effect, Switching technique

Paper type Research paper

1. Introduction

Computational methods based on classical continuum mechanics face particular problems whenever a discontinuity arises in the domain, as a fundamental assumption made in the theory is the continuity of the domain itself. The presence of a crack prevents the definition of the spatial derivatives appearing in the equations. Therefore, modelling of fracture problems, which is of great concern in many applications, has been a challenging issue for computational mechanics community for many years. An enormous amount of research

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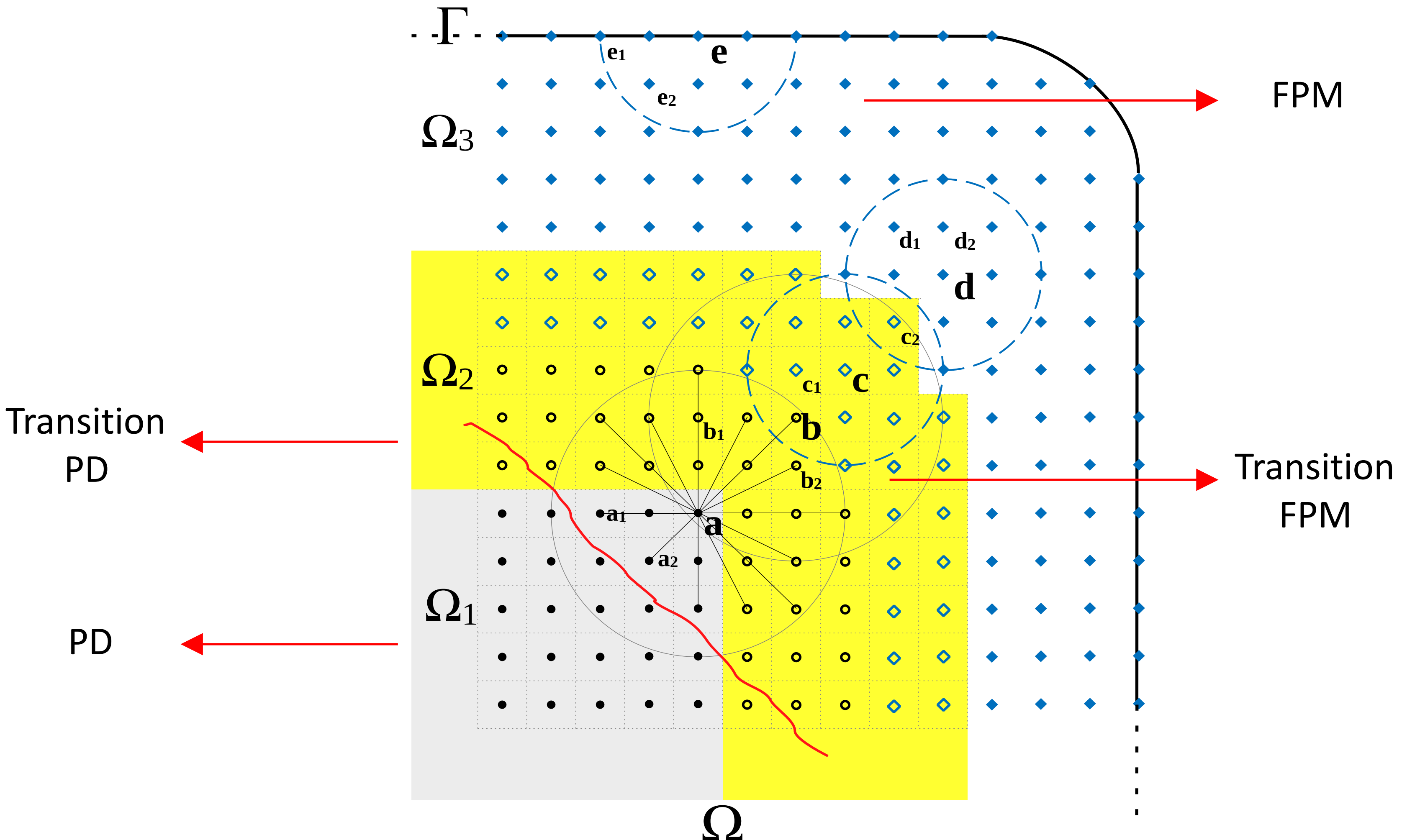
The coupling technique: the main idea



The coupling technique: the main idea

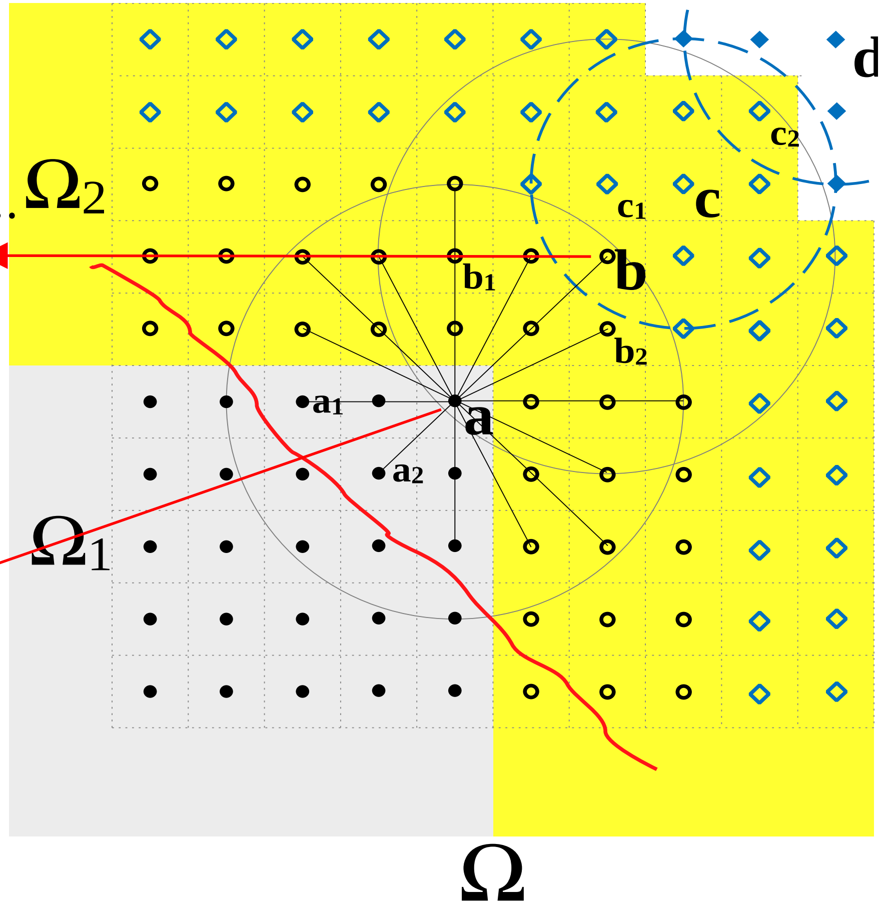
Layer b – Transition PD:

$$\rho \ddot{\mathbf{u}}_b^n = \cdots \mu_{ba}^n \hat{\mathbf{f}}_{ba}^n + \cdots + \mu_{bb_1}^n \hat{\mathbf{f}}_{bb_1}^n + \mu_{bb_2}^n \hat{\mathbf{f}}_{bb_2}^n + \cdots$$
$$+\mu_{bc}^n \hat{\mathbf{f}}_{bc}^n + \mu_{bc_1}^n \hat{\mathbf{f}}_{bc_1}^n + \mu_{bc_2}^n \hat{\mathbf{f}}_{bc_2}^n + \cdots \mathbf{b}_b^n$$

Layer a – PD:

$$\rho \ddot{\mathbf{u}}_a^n = \cdots \mu_{aa_1}^n \hat{\mathbf{f}}_{aa_1}^n + \mu_{aa_2}^n \hat{\mathbf{f}}_{aa_2}^n + \cdots$$
$$\mu_{ab}^n \hat{\mathbf{f}}_{ab}^n + \mu_{ab_1}^n \hat{\mathbf{f}}_{ab_1}^n + \mu_{ab_2}^n \hat{\mathbf{f}}_{ab_2}^n + \cdots + \mathbf{b}_a^n$$

$$\hat{\mathbf{f}}_{ij}^n = \mathbf{k}_{ij}^{11} \mathbf{u}_i^n + \mathbf{k}_{ij}^{21} \mathbf{u}_j^n$$



The coupling technique: the main idea

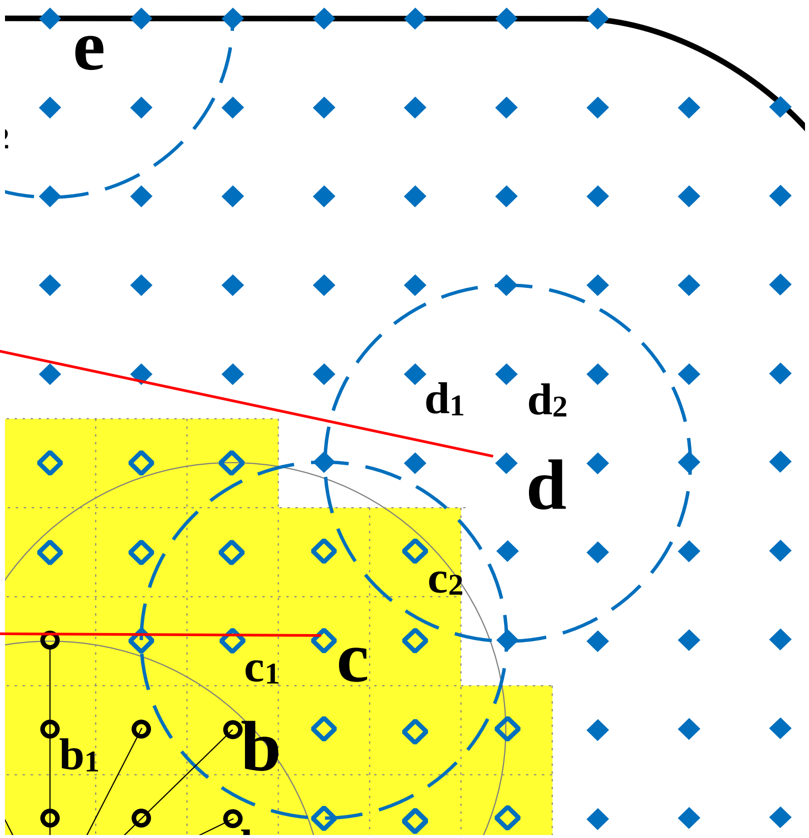
Layer d – FPM:

$$\rho\ddot{\mathbf{u}}_d^n = \cdots + \bar{\mathbf{f}}_{dd} + \bar{\mathbf{f}}_{dd_1} + \bar{\mathbf{f}}_{dd_2} \ldots \mathbf{b}_d^n$$

Layer c – Transition FPM:

$$\rho\ddot{\mathbf{u}}_c^n = \cdots + \bar{\mathbf{f}}_{cb} + \cdots + \bar{\mathbf{f}}_{cc} + \bar{\mathbf{f}}_{cc_1} + \bar{\mathbf{f}}_{cc_2} + \cdots \mathbf{b}_c^n$$

$$\bar{\mathbf{f}}_{ij} = (\mathbf{S}^{\mathrm{T}}\mathbf{D}\mathbf{S}\mathbf{N}_j)|x_i\mathbf{u}_j^n$$



The coupling technique: Patch test

Rigid body motion: Translation and rotation

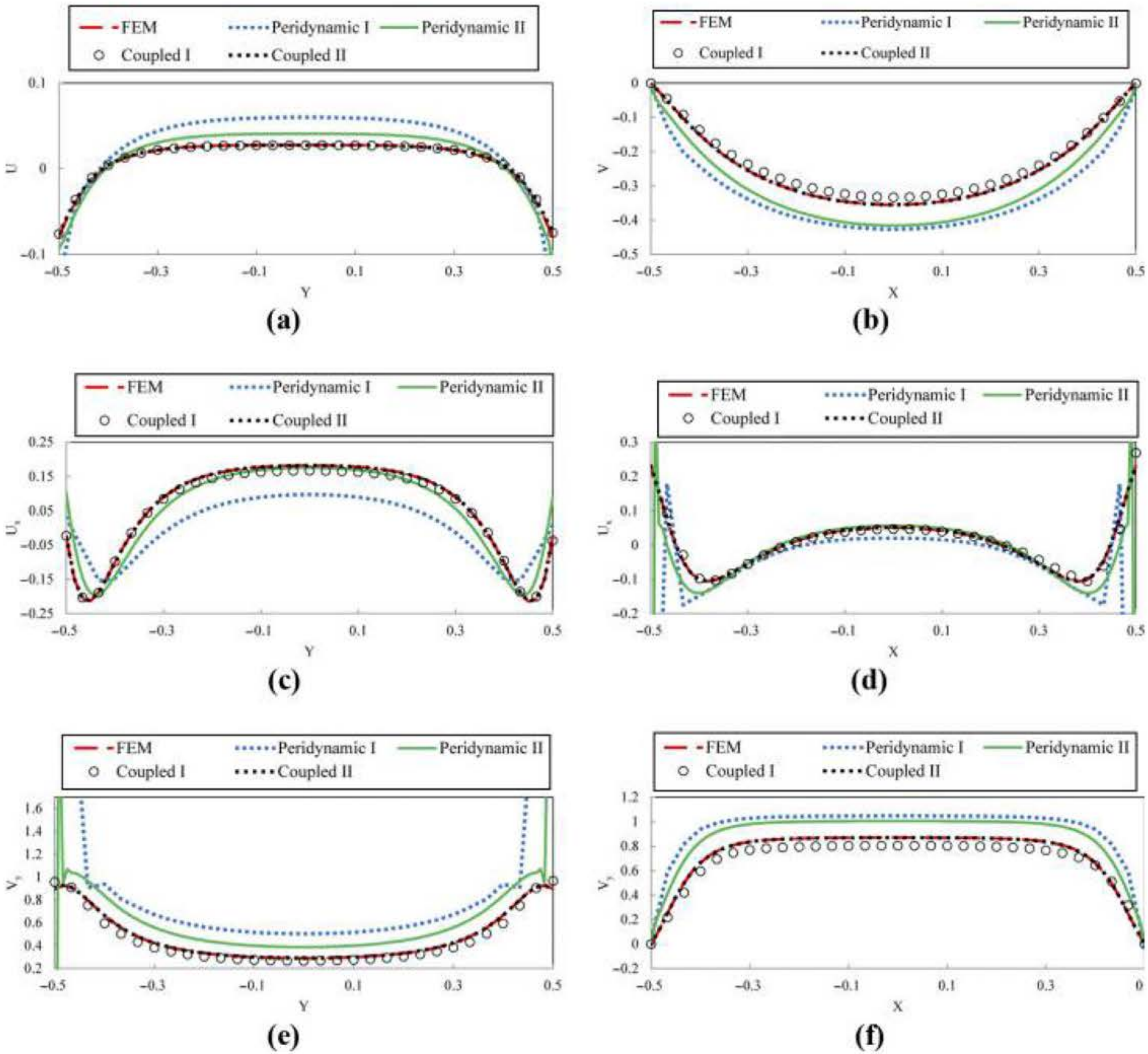
m	$\sqrt{2}$	2	3	4	5
Irregular discretization	8.54E-09	8.46E-09	8.27E-0.9	7.93E-0.9	7.41E-09
Regular discretization	1.52E-07	3.38E-08	2.20E-08	6.69E-09	6.32E-09

The results obtained for e_u in the ghost force test, rigid body translation, for regular and irregular discretization cases and different values of m ratio

→ Ghost forces are negligible

The results obtained for e_u in the ghost force test, rigid body rotation, for regular and irregular discretization cases and different values of m ratio

m	$\sqrt{2}$	2	3	4	5
Irregular discretization	1.72E-08	1.72E-08	1.71E-08	1.68E-08	1.64E-08
Regular discretization	1.56E-08	8.41E-09	1.57E-08	8.08E-09	2.07E-08



comparison of results obtained by different approaches: (a) U along L_1, (b) V along L_2, (c) U_x along L_1, (d) U_x along L_2, (e) V_y along L_1 and (f) V_y along L_2

Dynamic fracture analysis

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Dynamic fracture ar

Static configuratior

$t = t_0$

Adaptive configura

$t = t_0$

Acta Mech 228, 3581–3593 (2017)
DOI 10.1007/s00707-017-1894-4

CrossMark

ORIGINAL PAPER

Arman Shojaei · Farshid Mossaiby · Mirco Zaccariotto ·
Ugo Galvanetto

The meshless finite point method for transient elastodynamic problems

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Abstract In this paper, the application of the meshless finite point method (FPM) to solve elastodynamic problems through an explicit velocity–Verlet time integration method is investigated. Strong form-based methods, such as the FPM, are generally less stable and accurate in terms of satisfaction of Neumann boundary conditions than weak form-based methods. This is due to the fact that in such types of methods, Neumann boundary conditions must be imposed by a series of equations which are different from the governing equations in the problem domain. In this paper, keeping all the advantages of FPM in terms of simplicity and efficiency, a new simple strategy for proper satisfaction of Neumann boundary conditions in time for elastodynamic problems is investigated. The method is described in detail, and several numerical examples are presented. Moreover, the accuracy of the method with reference to the solution of some 3D problems is discussed.

1 Introduction

During the past few decades, the computational mechanics community has given specific attention to so-called mesh reduction methods. Thereupon, there is a fast growing interest in developing meshless (or mesh-free) methods as an alternative to conventional mesh-based methods such as the finite element method (FEM). Although FEM has been developed thoroughly and applied successfully to a variety of engineering problems, the task of mesh generation can be very costly and burdensome for three-dimensional problems especially those with complicated domains. The key idea of meshless methods is to provide numerical solutions on a set of arbitrarily distributed nodes instead of elements, and replacing meshing or re-meshing procedures with adding or eliminating nodes at desired parts. The reader may refer to [14,17] for development history of meshless methods. Depending on how equations are discretized, meshless methods can be classified into two major categories. The first category constitutes meshless methods based on weak form such as the element-free Galerkin method [3]. Most of them are only meshless in terms of the numerical approximation of field variables, and they are involved with numerical integration using a background mesh over the problem domain, which makes them computationally expensive and not "truly" meshless.

The second category is meshless methods based on the strong form such as the finite point method (FPM) [19]. These methods often use the point collocation method to satisfy the governing differential equations such

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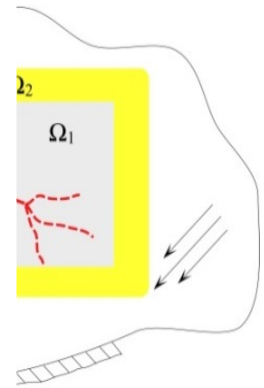
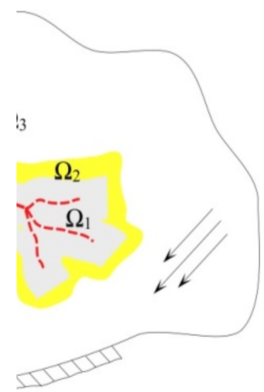




Dynamic fracture analysis: Domain partitioning

E

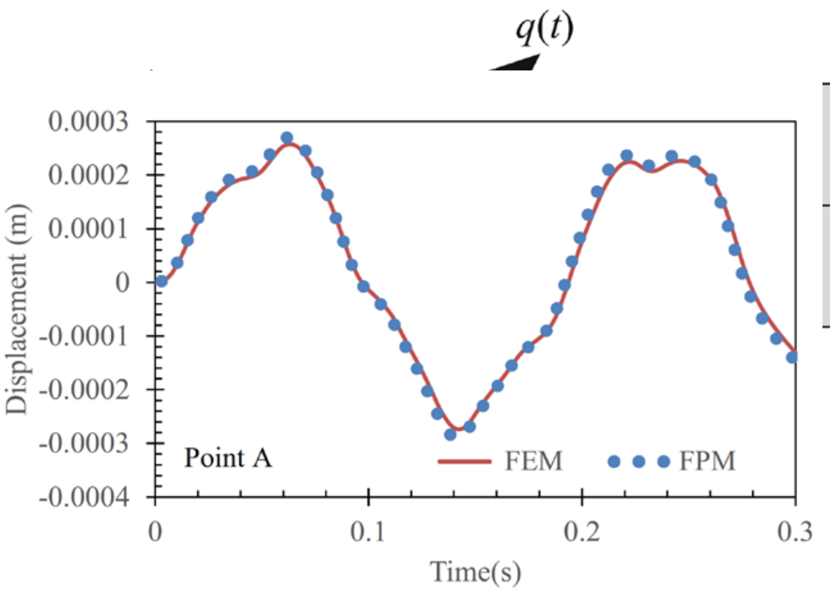
ν

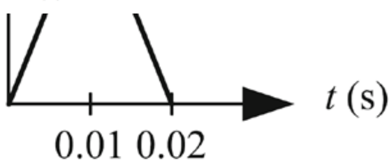
ρ

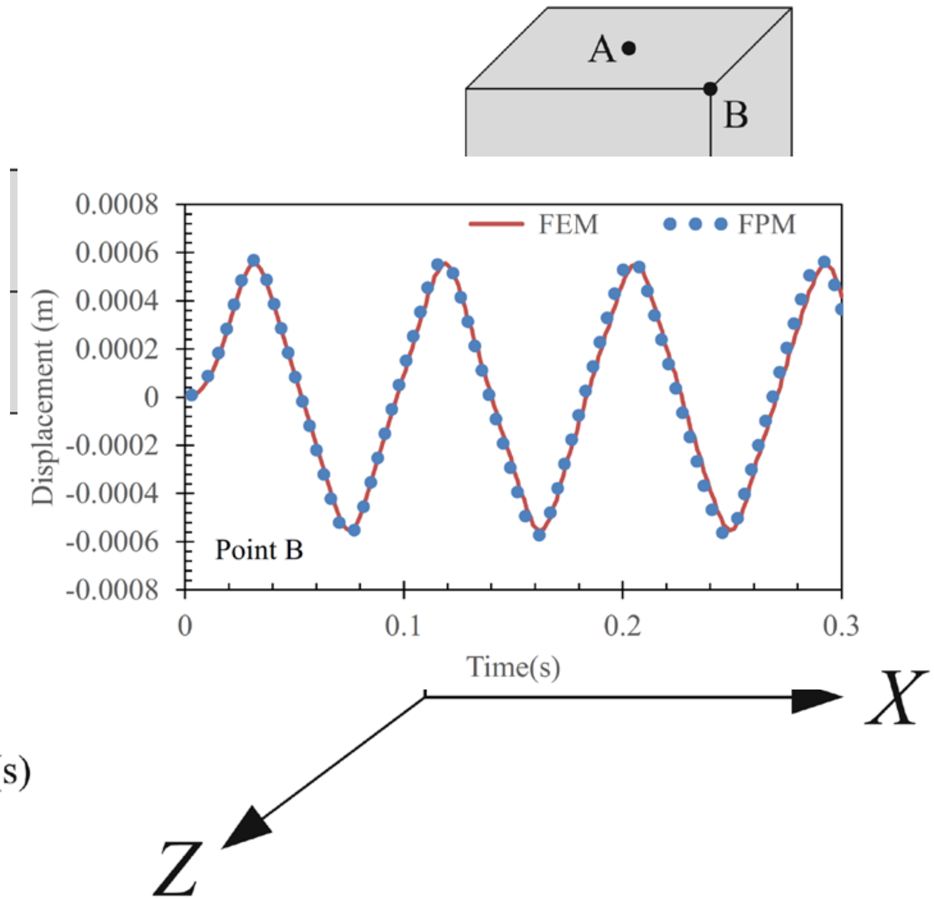
Δ

H

$D = 2\text{m}$

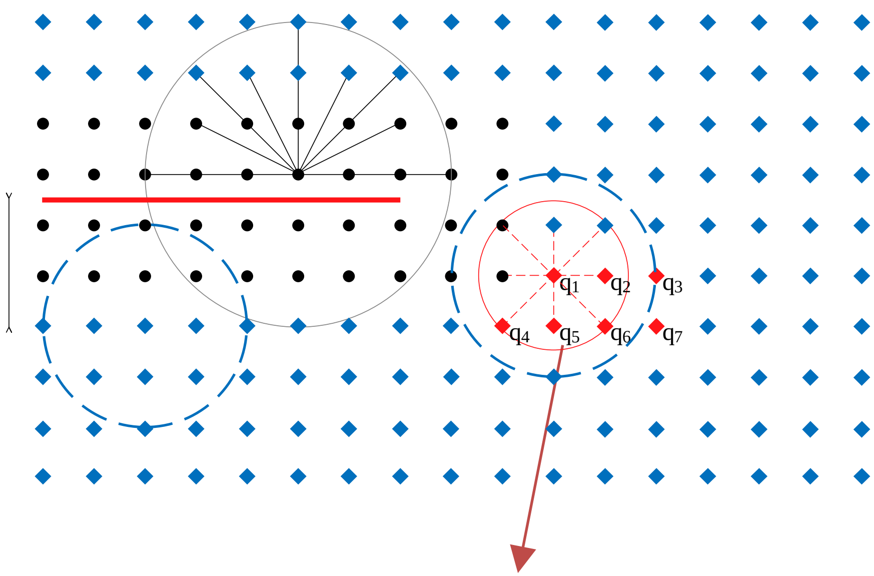


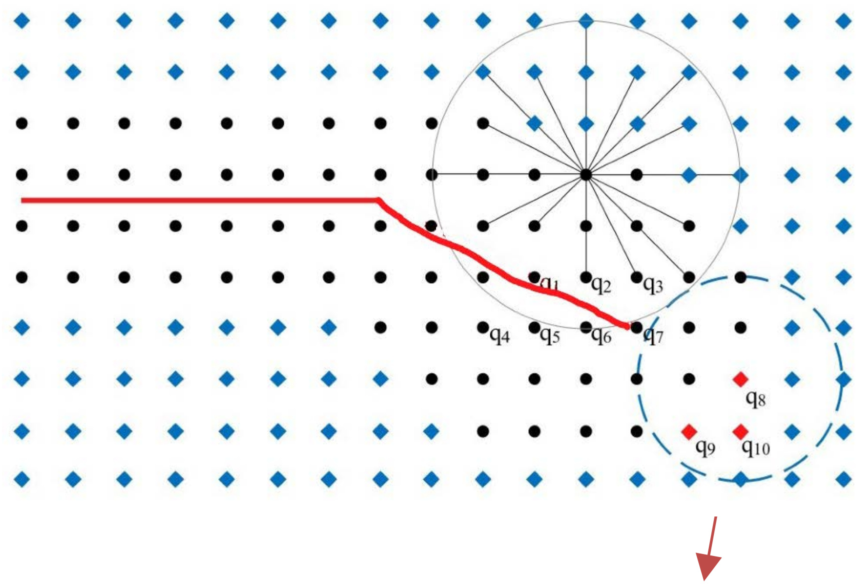




Dynamic fracture analysis: Domain partitioning

Adaptive switching





Monitoring cloud of critical nodes

New critical nodes

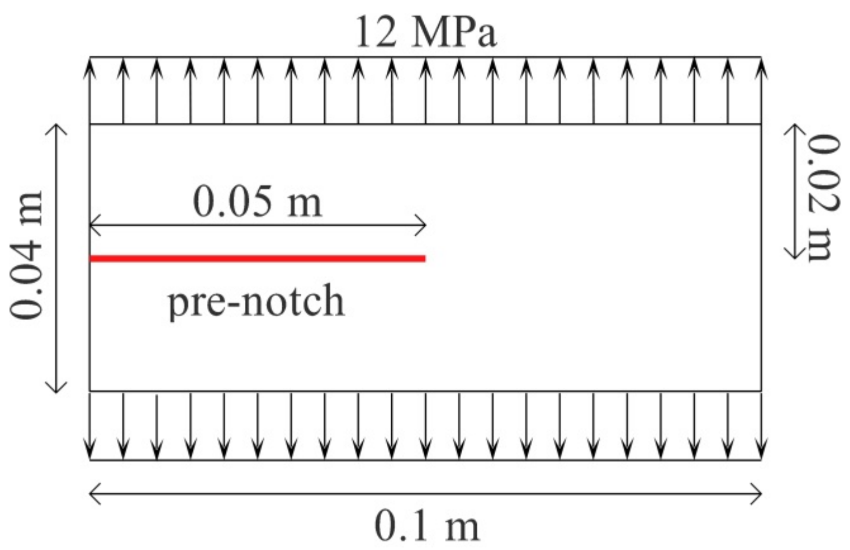
$$\chi s_0 \leq s'^{n}_{ij} < s_0, \qquad 0 \leq \chi \leq 1$$

Critical stretch

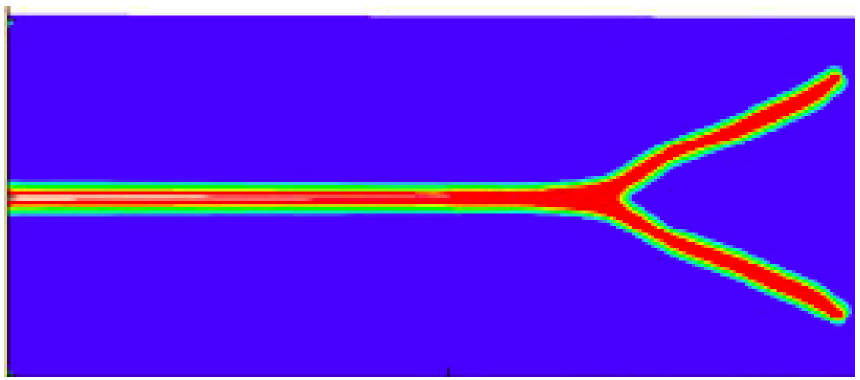
$$s'^{n}_{ij} = \frac{|\mathbf{u}_i^n - \mathbf{u}_j^n|}{|\mathbf{x}_{ij}|}$$

Dynamic fracture analysis: Benchmark examples

Pre-cracked plate:



Duran 50 glass
Time Duration: 46 μs

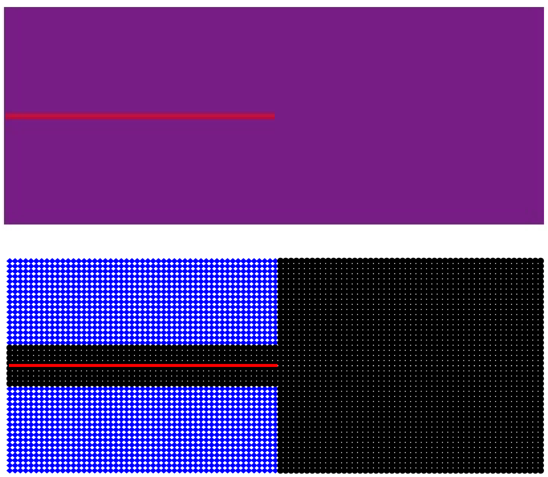


A PD-only model by Ha & Bobaru 2010 using 64,000 nodes

Ha, Y.D., Bobaru, F.: Studies of dynamic crack propagation and crack branching with peridynamics. Int. J. Fract. 162, 229–244 (2010).

Dynamic fracture analysis: Benchmark examples

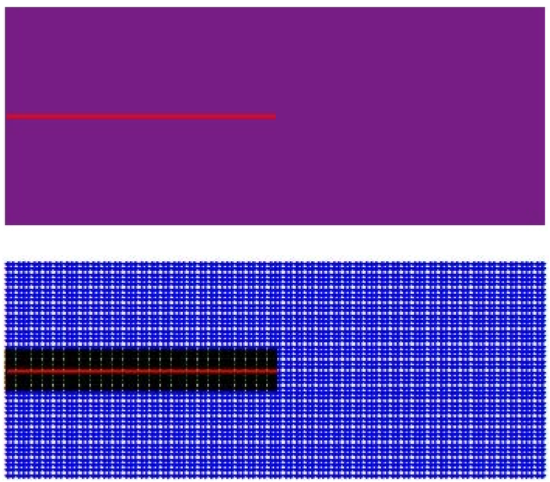
Pre-cracked plate:



Model I

Static Partitioning

Δ=1 mm; 4242 Nodes

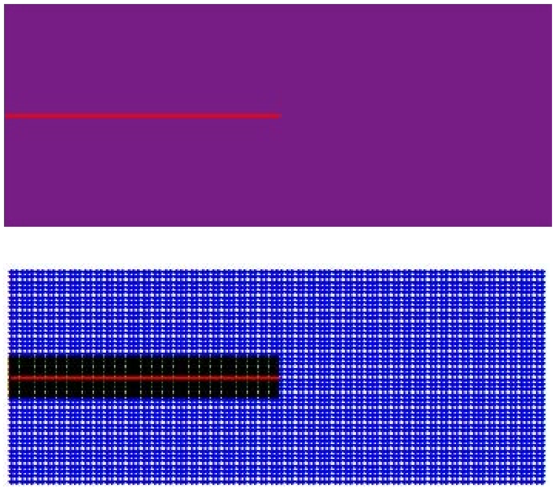


Model II

Adaptive Partitioning

Δ=1 mm; 4242 Nodes

χ=0.6



Model III

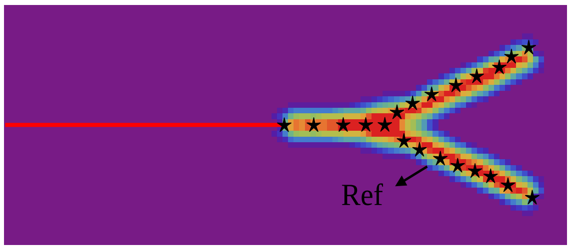
Adaptive Partitioning

Δ=1 mm; 4242 Nodes

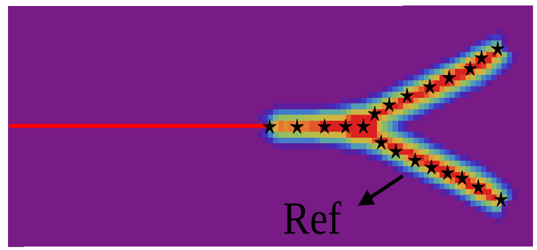
χ=0.8

Dynamic fracture analysis: Benchmark examples

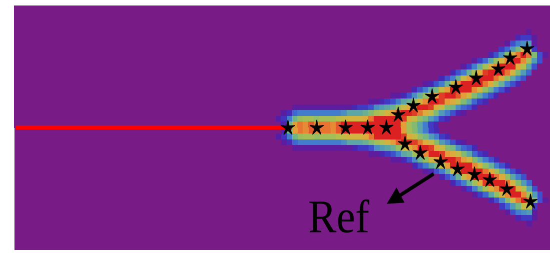
Pre-cracked plate:



Model I



Model II



Model III

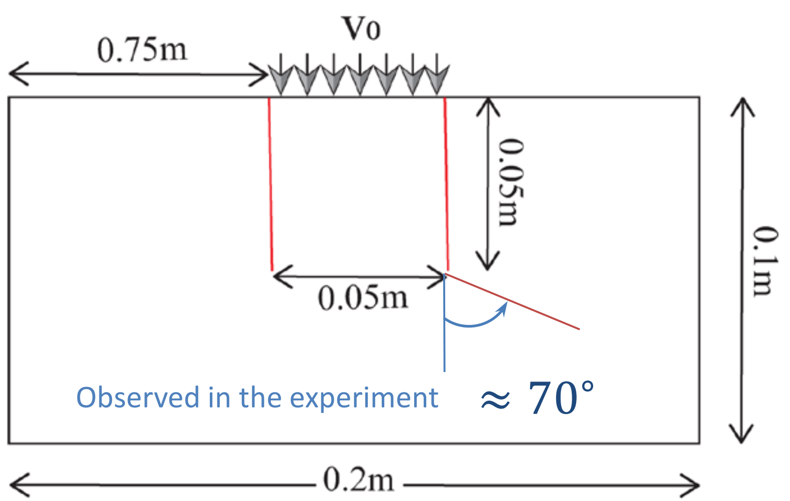
Model	Portion of Peridynamic nodes at at t=0	Portion of Peridynamic nodes at t=46 µs	CPU Time (s)
Model I	59 %	59 %	2362.78
Model II	9 %	29.4 %	1256.80
Model III	9 %	25.69 %	1207.52

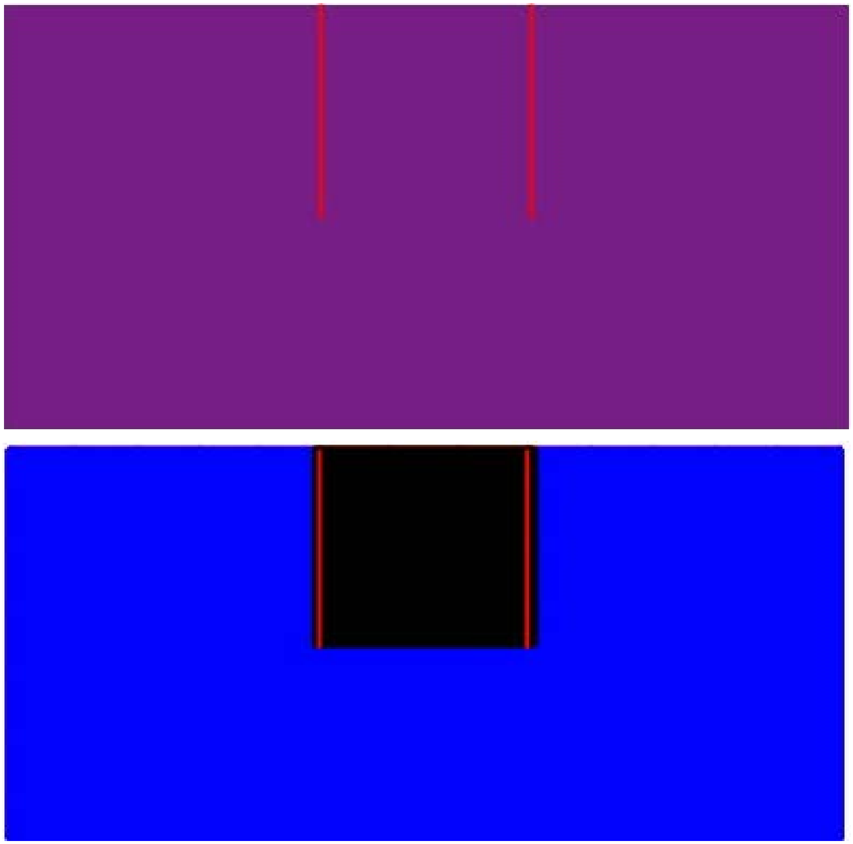
System Properties: Intel® Core™ i7-3770 CPU @ 3.40 GHz ;Ram: 6 GB; Windows 7 Professional

Dynamic fracture analysis: Benchmark examples

32m/s

Kalthoff-Winkler's Experiment:





Kalthoff, J. F. (2000). Modes of dynamic shear failure in solids. *International Journal of Fracture*, *101*(1/2), 1–31

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International Journal of Mechanical Sciences 119 (2016) 419–431

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Dynamic fracture a

Kalthoff-Winkler's

A coupled meshless finite point/Peridynamic method for 2D dynamic fracture analysis

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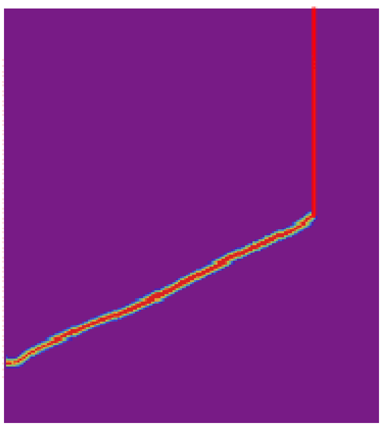
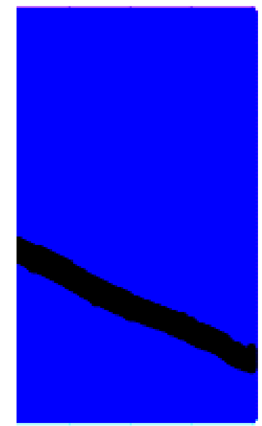
Coupling
Switching
Crack propagation

ABSTRACT

In this paper, a new coupled meshless method for 2D transient elastodynamic problems involving dynamic crack propagation is developed. The method is based on an efficient coupling between the finite point method (FPM) and a discretized form of Peridynamics. The solution domain is partitioned in three parts: one discretized by Peridynamics, one by FPM, and a transition part discretized by both methods where the switching between the two approaches is performed. The coupling adopts a local/nonlocal framework that benefits from the full advantages of both methods. The parts of domain where cracks either exist or are likely to propagate are described by Peridynamics; the remaining part of the domain is described by FPM that requires less computational effort. The capabilities of the proposed approach are demonstrated by means of the solution of dynamic problems including dynamic fracture as well as a ghost force test.





1. Introduction

Accurate modeling of damage and fracture phenomena is still an open issue for the community of computational mechanics [1]. The finite element method (FEM) is the most popular computational technique for structural computations. It is robust and has been thoroughly developed for static, dynamic, linear and nonlinear mechanical systems. However, modeling of problems with evolving discontinuities by conventional FEM models is a challenging issue, and special techniques need to be devised. One of the main viable options to cope with moving cracks using conventional FEM is to update the mesh during each step of analysis in such a way that the element edges coincide with cracks during all steps. Even though re-meshing approaches have so far been developed to address these limitations, for instance in [2], they are often applicable only to 2D cases. Moreover, such re-meshing processes are affected by numerical difficulties, complexity in computer programing and often lead to degradation of solution accuracy [3].

The introduction of interface elements coupled with Cohesive Zone Models (CZM) [4–6] provides the possibility of separation of adjacent elements between which the interface elements have been introduced. These methods have been applied with success, yet their application is limited to the cases in which the crack path is known in advance since the crack path is required to align along the edge of elements. Therefore, still adaptive re-meshing techniques have been proposed by researchers for the use of interface elements, as in [7], to resolve such difficulties. The eXtended finite element method (XFEM) was introduced in [8], it does not need any a-priory knowledge of the crack path and can robustly track crack paths without re-meshing [9–12]. XFEM incorporates discontinuity in the displacement field along the crack path regardless of crack location; furthermore, it exploits the partition of the unity property of finite elements [13]. However, the formulation of XFEM with increasing number of cracks and branches as well as its extension to three-dimensional cases becomes cumbersome. In addition, both CZM and XFEM models require extra damage criteria such as angle of propagation and stress state around the crack tip; furthermore, XFEM suffers from the absence of reliable branching criteria [14,15].

With the aim of eliminating re-meshing techniques, over the past decades, meshless methods have attracted the attention of many researchers. Unlike conventional FEM, in meshless methods the adaptive scheme can be easily developed as there is no mesh and thus no so called *a priory* connectivity is required between the nodes. This fact culminates in providing a flexible computational tool, and particularly in the case of crack propagation, the burdensome re-meshing required by conventional FEM models is avoided. Fundamentally in meshless methods, only a scattered set of nodal points is required, not necessarily a structured mesh, to represent the domain of interest. Such an appealing feature presents significant implications for modeling crack propagation. A variety of meshless methods have been

PD Nodes: 1

17.5%

: 80802

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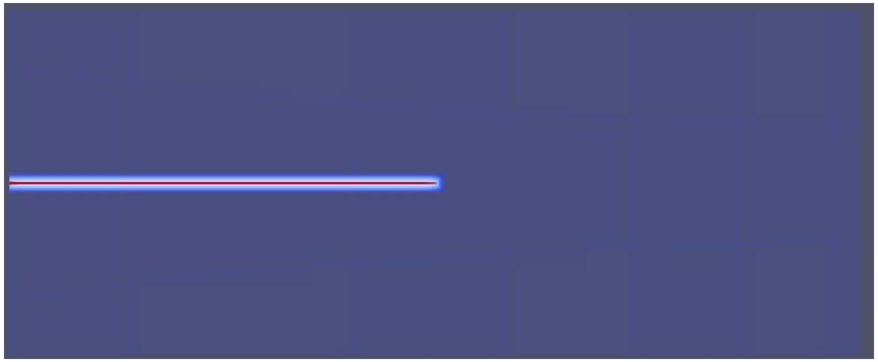
Conclusions and perspectives

Conclusions and perspectives: Remarks

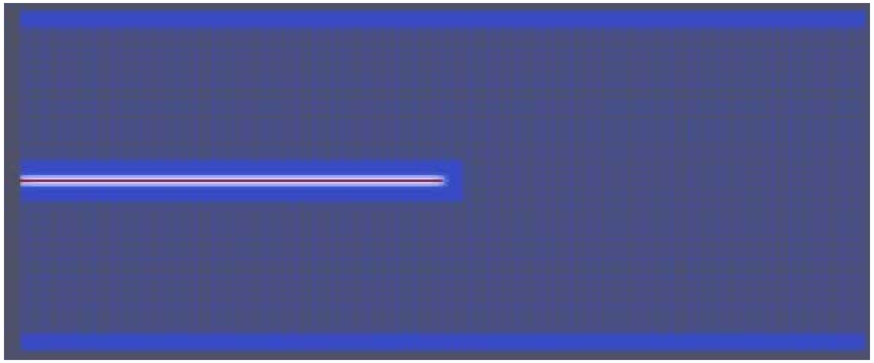
- A new coupling technique to couple a Peridynamic meshless method with a conventional meshless method is introduced.
- The coupling is done in a complete meshless style preserving the originality of both formulations.
- The coupling is done so that no ghost forces emerge in the transition part.
- The way of coupling is simple, and it does not introduce any numerical artifact.
- The coupling technique is capable of being used in an adaptive style
- The coupled method is capable to reproduce the solution of a Peridynamic-only model more efficiently.

Conclusions and perspectives: Perspectives

- To extend the work to 3D problems through a rebuts software implementation
- To equip the Finite Element Method (FEM), inspired by the present study, for dynamic failure analysis; to be useful for being implemented in a commercial software (ABAQUS)



Dual adaptive coupling of FEM and PD



Multiple branching

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Conclusions and p

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OpenCL implementation of a high performance 3D Peridynamic model on graphics accelerators

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ABSTRACT

Parallel processing is one of the major trends in the computational mechanics community. Due to inherent limitations in processor design, manufacturers have shifted towards the multi- and many-core architectures. The graphics processing units (GPUs) are gaining more and more popularity due to high availability and processing power as well as maturity of development tools and community experience. In this research we describe a rather general approach to using OpenCL implementation of 3D Peridynamics model on GPU platform. Peridynamics is a non-local continuum theory for describing the behavior of material used especially when damage and crack nucleation or propagation is of interest. The steps taken for developing an OpenMP code from the serial one as well as the comparison between OpenCL and OpenMP codes are provided. Optimization techniques and their effects on the performance of the code are described. The implementations are tested on some 3D benchmarks with hundred of thousands to millions of nodes. The behavior of codes in terms of being memory or compute bound are analyzed. In all test cases reported, the OpenCL implementation consistently outperforms serial and OpenMP ones and paves the road for the development of high performance Peridynamics codes.

1. Introduction

Failure mechanism of solid materials and structures is a long standing problem in computational mechanics. In particular, accurate modeling of damage and fracture phenomena, including static and dynamic crack propagation, is still an active and open challenge among researchers. The main difficulty inherent in such problem arises from the fact that the crack nucleation and propagation in materials cannot be well predicted by the majority of currently available computational techniques based on classical continuum theory of mechanics. The classical theory employs partial differential equations assuming that a body remains continuous as it deforms. Therefore, the spatial derivatives involved in these equations do not exist on a crack tip; moreover, they will become undefined or ill-suited where the displacement field is discontinuous [1]. In this regard, a variety of remedies have been introduced in the literature to equip available computational methods based on the classical theory; for instance, in [2–7]. Nonetheless, the majority of these techniques still suffer from unsatisfactory accuracy and low efficiency when they deal with complicated problems, such as three-dimensional propagation of cracks in bodies under complex loading conditions [8,9].

The Peridynamic theory was first introduced by Silling in [10] considered to be an alternative and promising non-local theory of solid mechanics that is formulated suitably for discontinuous problems such as crack propagation. Peridynamics

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Thank you very much

Any Question?