Adaptive grid refinement and scaling techniques applied to peridynamics

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Equation of motion

- For a given body $R_0$, the equation of motion of an infinitesimal particle of material is defined by means of the following expression:

$$\rho \ddot{u}(x_i, t) = \nabla [\sigma] + b(x_i, t)$$

$$\rho \ddot{u}(x_i, t) = \int_{H_{x_i}} f(u(x_j, t) - u(x_i, t), x_j - x_i) \, dV_j + b(x_i, t)$$

$$H_{x_i} = \{ x_j \in R_0 : \| x_j - x_i \| \leq \delta \} \quad \rightarrow \quad \text{HORIZON}$$

$$f(\eta, \xi) = f(\eta, \xi) \frac{\xi + \eta}{\| \xi + \eta \|} \quad \rightarrow \quad \text{PAIRWISE FORCE FUNCTION}$$

$u \rightarrow$ displacement vector field
$\rho \rightarrow$ mass density
$b \rightarrow$ body force density
$\xi = x_j - x_i \rightarrow$ initial relative position
$\eta = u_j - u_i \rightarrow$ relative displacement

Images by W. Liu and J.W. Honget
Constitutive law

- A brittle elastic material is modeled adopting the constitutive law called PMB (Prototype Microelastic Brittle). The scalar pairwise force function takes the form:

\[ f(\eta, \xi) = \mu(\xi) \cdot c \cdot s \]

\[ s = \frac{||\xi + \eta|| - ||\xi||}{||\xi||} \quad \text{stretch of the bond} \]

\[ c \propto \frac{E}{\delta^3} \quad \text{micromodulus} \]

\[ \mu(\xi) = \begin{cases} 
1 & \text{if } s < s_0 \propto \sqrt{G_0/E\delta} \\
0 & \text{otherwise} 
\end{cases} \]

\[ E := \text{Young's modulus} \]
\[ G_0 := \text{Fracture energy} \]
Damage definition

- It is possible to define a non-ambiguous state of **material damage** at every point $x_i$ of the body as:

$$\varphi(x_i) = \frac{\text{broken bonds}}{\text{initial bonds}}$$

- $\varphi(x_i) = 0$ means **undamaged state**
- $\varphi(x_i) = 1$ means **all broken bonds**

Images by ing. M. Duzzi
Some remarks about the numerical discretization

- Mesh-free method with a uniform structured grid and the Gauss quadrature mid-point space integration solved through the Velocity-Verlet explicit scheme:

\[
\rho \ddot{u}_i^n = \sum_j f(u_j^n - u_i^n, x_j - x_i)V_j \beta_j + b_i^n, \quad \forall \ x_j \in H_{x_i}
\]

\[
\rho \ddot{u}_i^n = \sum_k f(u_k^n - u_i^n, x_k - x_i') \beta_i \Delta V_k - \sum_j f(u_j^n - u_i^n, x_j - x_i) \beta_j \Delta V_j + b_i^n, \forall \ x_k \in H'_{x_i}, \forall \ x_j \in H_{x_i}
\]

\[m \text{ ratio} = \delta / \Delta x\]
AGRS

- The adaptive refinement and scaling allows to reduce in automatic mode (by means of a trigger) both the grid spacing and the horizon only in the regions of interest, as in the proximity of the crack tips during their propagation:

\[
\delta \rightarrow \text{convergence (} \delta \downarrow, m=\text{constant)}
\]

- Selected coarse node by trigger

  - based on potential energy (suggested in literature)
    \[
    W(x_i) \geq W_{\text{threshold}}
    \]
  - based on damage state, introduced by us
    \[
    \Delta \phi(x_i) = \phi(x_i) - \phi_0(x_i) > 0
    \]
    \[\phi_0(x_i) \text{ is the initial damage state of the nodes}\]
How the AGRS works

AGRS starts after an appropriate time determined by carrying out a preliminary analysis on the coarse grid.

Activation AGR in previous step
- Nodes generation with visibility criterion
- Updating the nodal areas and horizon
- Interpolation physical quantities

Is $\Delta \phi > 0$ of any node of the coarse grid?

Was node refined in previous step?

Activation AGR
- Nodes generation with visibility criterion
- Updating the nodal areas and horizon
- Interpolation physical quantities
Static Test

- A 2D static elastic linear problem is addressed through the comparison of the numerical peridynamic solution with the analytical solution of classic theory of mechanics:

\[ u_X = (X, Y = 0) = \frac{p}{E} X \]

\[ u_Y = (X = 0, Y) = -\nu \frac{p}{E} Y \]

- Example of 2\textsuperscript{nd} level of refinement and scaling applied:
Static Test

- The PD solution between Scaling and Scaling & Dual-horizon formulation are compared for the 1\textsuperscript{st} level of refinement/scaling:
Static Test

- The PD solution between Scaling and Scaling & Dual-horizon formulation are compared for the 3rd level of refinement/scaling:
The comparison between the solutions obtained by applying different levels of refinement/scaling highlights that higher levels of refinement do not affect the result:

\[ L_2 \text{ error} = \frac{1}{q} \sqrt{\frac{\sum_{i=1}^{j} (u_i^{PD} - u_i^{analytical})^2}{\sum_{i=1}^{j} (u_i^{PD})^2}} \]

The convergence study on m ratio shows as the rate of convergence of the refined region is higher than the coarse one:
Addressing grid sensitivity in peridynamics

• Problem of a 2D pre-cracked square plate subjected to a traction load:
  - The grid is rotated with respect to the direction of pre-crack line
  - Quasi-Static and Dynamic analysis
Addressing grid sensitivity in peridynamics

- Waves propagation regarding the dynamic case of the grid rotated of 10° with \( m = 3 \):
Addressing grid sensitivity in peridynamics

- Crack paths obtained with different rotated grids with $m = 3$, dynamic cases:

- Crack paths obtained with different rotated grids with $m = 3$, quasi-static cases:
Numerical explanation of grid sensitivity

- This type of space discretization introduces an anisotropy on the damage state of the node, namely an anisotropy on the energy required to break the bonds along a specific direction:

\[ \phi = \frac{\text{broken bonds}}{\text{initial bonds}} \]

Example with \( m \) ratio = 3

\[
\begin{align*}
\phi_0 &= 0.38 \\
\phi_{15} &= 0.48
\end{align*}
\]
Numerical explanation of grid sensitivity

- The directions of minimum energy required to break the bonds along a specific direction match the directions of the bonds:

Example with $m$ ratio = 3
Numerical explanation of grid sensitivity

- When the $m$ ratio increases both the number of the minimum energy directions increases and the gap energy between them and the other directions reduces:

- Regions in which the bond directions remain «rare»
Addressing grid sensitivity in peridynamics

• With reference to the worst case of grid rotated of 10 degree, it is possible to see as an increase of $m$ ratio from 3 to 6-7 is enough to eliminate the dependence of crack propagation on grid orientation:
Addressing grid sensitivity with AGRS

- Application of the 1\textsuperscript{st} level of AGRS when the grid is rotated of 10°, the horizon is kept constant:

- **Initial Grid**: 15,600 nodes
  \[ \Delta x = 2 \text{ mm} \]
  \[ m = 3 \]

- **Refined region**: \[ \Delta x = 1 \text{ mm} \]
  \[ m = 6 \]
Addressing grid sensitivity with AGRS

- Application of the 2\textsuperscript{nd} level of AGRS when the grid is rotated of 10°, the \textbf{horizon shrinks}:

- Initial Grid: 15,600 nodes
\[ \Delta x = 2 \text{ mm} \]
\[ m = 3 \]

- Uniform Refined region: \[ \Delta x = 0.5 \text{ mm} \]
\[ m = 6 \]

- Refined region: \[ \Delta x = 0.5 \text{ mm} \]
\[ m = 6 \]

CPU time: 16.09 hour

CPU time: 3.52 hour
Benchmark problem: Kalthoff-Winkler’s experiment

- **Setup experiment:**

  - **Material 18Ni1900:**  \( E = 190 \text{ GPa} , \)  
    \( \rho = 8000 \text{ kg/m}^3 \)  
    \( G_0 = 22170 \text{ J/m}^2 \)
  
  - **Simulation:**  \( t_{\text{tot}} = 52 \mu\text{s} (\Delta t_{\text{min}} = 20 \text{ ns}) \)
  
  - **Initial Grid:**  5,000 nodes  
    \( \Delta x = 2 \text{ mm} \)  
    \( \delta = 6 \text{ mm} \)
  
  - **Energy Trigger:**  \( W \geq 0.7 W_{\text{max}} \)
  
  - **Damage Trigger:**  \( \Delta \phi \geq 0 \)
  
  - **\( \delta \)-convergence (\( m=3 \))**

Benchmark problem: Kalthoff-Winkler’s experiment

Uniform coarse grid ( $\Delta x = 2\,mm$)

Uniform refined grid ( $\Delta x = 0.5\,mm$)

Adaptive grid with 2$^{nd}$ level of refinement
( $\Delta x_0 = 2\,mm$, $\Delta x_2 = 0.5\,mm$)

Adaptive model is able to capture the right angle of approximately $70^\circ$
3D Adaptive grid refinement/scaling

- Crack branching of pre-cracked glass plate under traction, the 1\textsuperscript{st} level of AGRS is applied in a 3D model:

Morphology observed experimentally
Activities related to my Ph.D

• Optimization of the pre-existent Matlab codes in the context of dynamic simulations with tools such as MEX files:

  - Optimization:
    - 16,000 nodes: ≈ 9 hours
    - 120,000 nodes: ≈ 5 hours

• Implementation of codes to import a general 3D mesh in Matlab environment
Publications


Duzzi M., Zaccariotto M., Dipasquale D., Galvanetto U. (2014), A Concurrent Multiscale Model to Predict Crack Propagation in Nanocomposite Materials with the Peridynamic Theory. Poster will be presented at International NanotechItaly2014 Conference, Italy


Publications


• Participation at 11\textsuperscript{th} World Congress on Computational Mechanics, Barcelona (20-25/07/2014)

• Participation at International CAE Conference, Pacengo del Garda (27-28/10/2014)

• Participation at 4\textsuperscript{th} International Conference on Computational Modeling of Fracture and Failure of Materials and Structures, Paris (02-05/06/2015)

• I will participate in IMECE International Mechanical Engineering Congress & Exposition, Phoenix, Arizona, USA (11-17/11/2016)
Conclusion

- Development of a robust algorithm to implement AGRS on peridynamics through the introduction of a trigger based on damage state of the nodes

- Development of both 2D/3D codes to implement AGRS with Matlab

- Optimization of the pre-existent Matlab codes in the context of dynamic simulations

- Comparison of different peridynamic formulations (Scaling and Dual-horizon) by means of both static and dynamic analysis

- Addressing dependence of crack propagation on grid orientation

- Validation of the numerical results obtained with other methods/experimental results
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