Università degli Studi di Padova

Peridynamic modelling of crack propagation in inelastic materials for aerospace applications

Atefeh Pirzadeh - 36th Cycle

Supervisor: Prof. Ugo Galvanetto

Co-supervisors: Prof. Mirco Zaccariotto, Dr. Federico Dalla Barba

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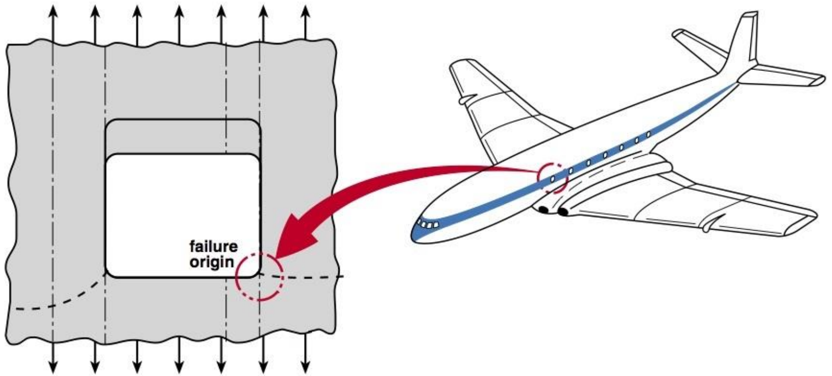
Materials for aerospace applications

- The most used commercial aerospace structural materials are aluminum alloys, titanium alloys, high-strength steels, and composites.

- Aerospace structures are subjected to different kinds of load, which may cause them to experience failure in some parts.

- Analysis of crack propagation in solid materials is very relevant to practical applications.



Motivation for Peridynamic

Classical theory of continuum mechanics

Some limitations for studying crack propagation

- It has partial derivatives with respect to spatial coordinates, which are undefined along the cracks
- To compensate this weakness, the problem is redefined along the cracks
- Not appropriate for spontaneous cracks
- Some assumptions like the time crack initiates and crack growth speed

Simulating crack propagation is a challenging task

Finite Element Method (FEM), Extended FEM (XFEM) and Meshless Methods

Peridynamic is a nonlocal continuum mechanics theory (developed by Silling in 2000).

- Modeling problems with singularities like cracks
- Appropriate for large number of cracks
- Integration is used instead of differentiation

Peridynamic Formulation

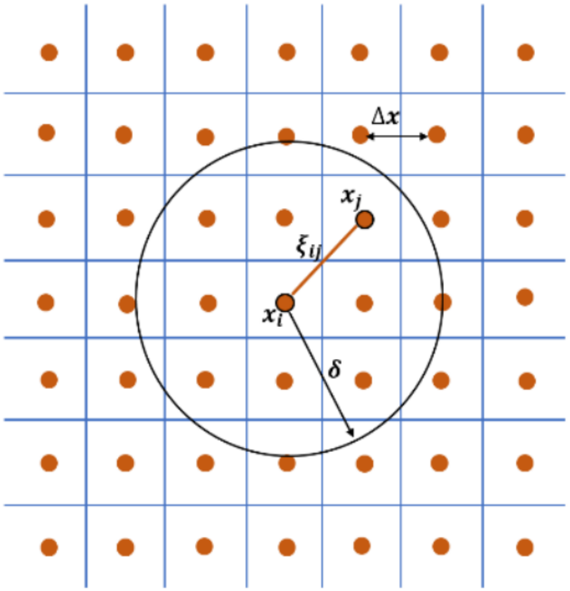
$$\rho(\boldsymbol{x}_i)\ddot{\boldsymbol{u}}_{n+1}(\boldsymbol{x}_i) = \sum_j \left(\underline{\boldsymbol{T}}_n[\boldsymbol{x}_i](\boldsymbol{x}_j - \boldsymbol{x}_i) - \underline{\boldsymbol{T}}_n[\boldsymbol{x}_j](\boldsymbol{x}_i - \boldsymbol{x}_j)\right) \Delta V_{\boldsymbol{x}_{ij}} + \boldsymbol{b}_n(\boldsymbol{x}_i)$$

Bond-based

- The force in each bond does not depend on other bonds deformation
- Can not enforce incompressible shear deformation

Ordinary-State-based

- The force of each bond depends on all the bonds inside the horizon

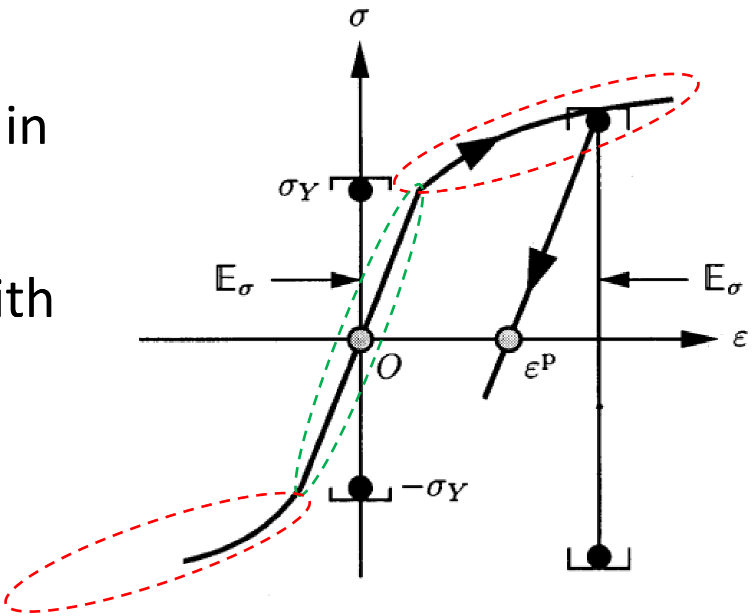


Motivation for Elastoplastic Approach

Most of the Peridynamic based models are used for fracture analysis in **Elastic Domain**.

Before crack initiation, most materials experience deformation in **Plastic Domain**.

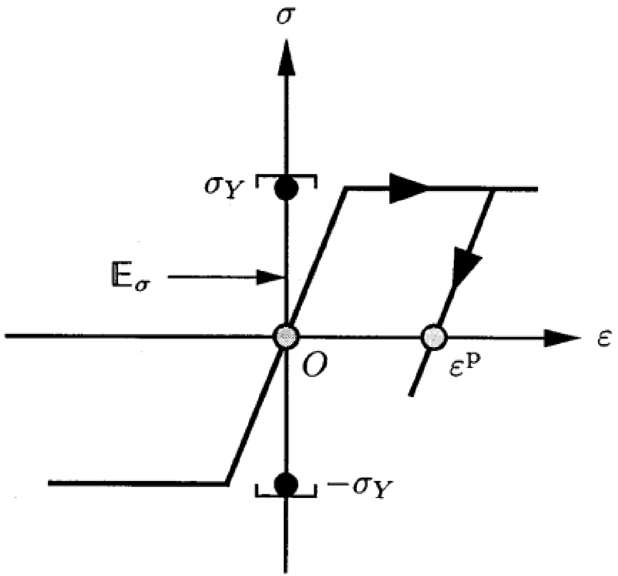
Equipping the bonds of **OSB-PD** with an **Elastoplastic** constitutive law is the objective of this research.



Motivation for Elastoplastic Approach

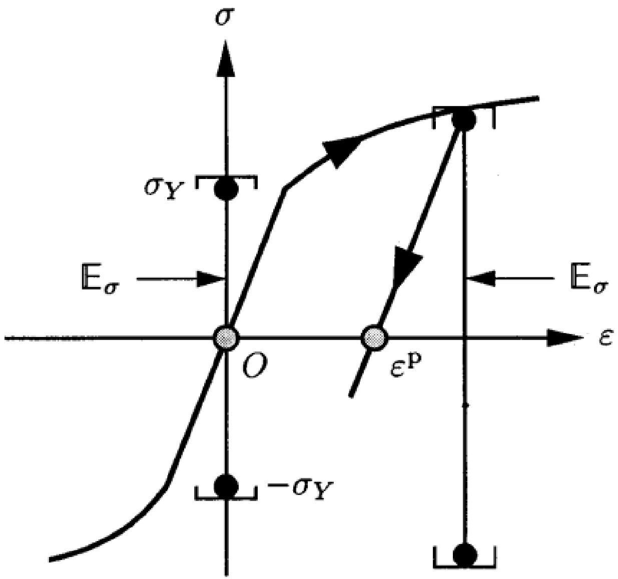
Materials with **perfect plasticity**:

Closure of the elastic range (E_σ) remains unchanged.



Materials with **strain hardening**:

Closure of the elastic range (E_σ) expands with the amount of slip in plastic domain.



Elastoplastic Analysis

Kuhn-Tucker condition:

$$\lambda f(\sigma,\alpha,q)=0, \quad \lambda=\left|\dot{\varepsilon}_p\right|$$

Consistency condition:

$$\lambda \dot{f}(\sigma,\alpha,q)=0$$

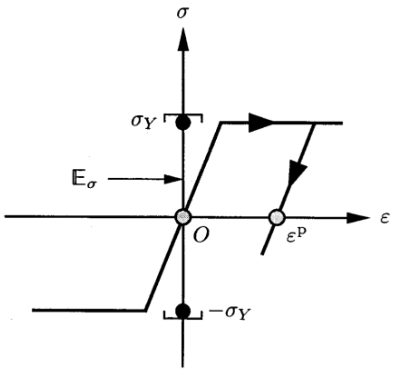
Elastic stress-strain relationship:

$$\sigma=E(\varepsilon-\varepsilon_p)$$

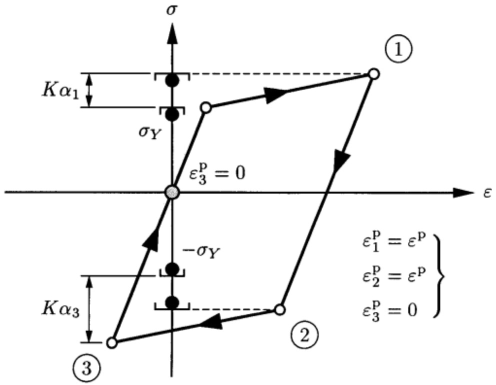
Yield condition:

$$f(\sigma,\alpha,q)=|\sigma-q|-(\sigma_Y+K\alpha)\leq 0$$

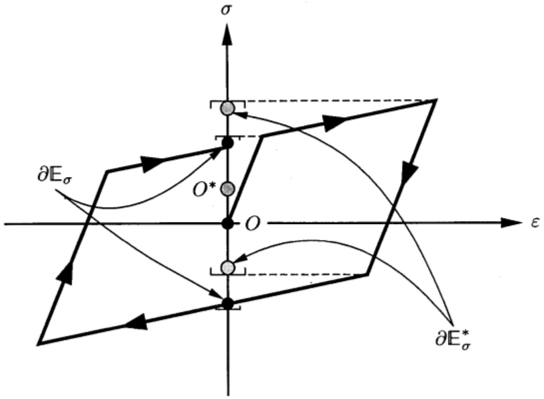
$$\begin{cases} \dot{\alpha} = \dot{\varepsilon}^p \operatorname{sign}(\sigma - q) & \text{Isotropic hardening} \\ \dot{q} = \dot{\varepsilon}^p H & \text{Kinematic hardening} \end{cases}$$



Perfect plasticity



Isotropic hardening

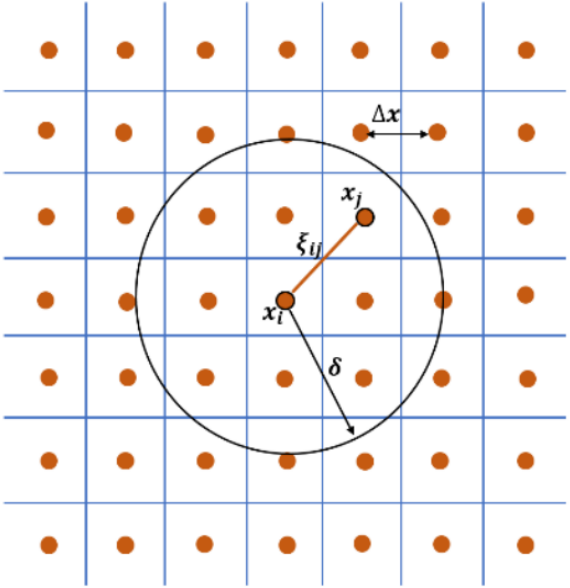


Kinematic hardening

State-based Elastoplastic Analysis

In J2 plasticity, the von-Mises yield criterion is used based on maximum deviatoric strain energy. for 3D case:

$$f(\underline{t}_{trial}^{d}) = \frac{\left\| \underline{t}_{trial}^{d} \right\|^2}{2} - \psi_0 \equiv \quad f(\sigma,\alpha,q) = |\sigma - q| - (\sigma_Y + K\alpha) \leq 0$$



Yield condition:

$$if \;\; f(\underline{t}_{trial}^{d}) \leq 0 \;\Rightarrow\; elastic\ step \;\; (\Delta\lambda = 0) \Rightarrow \begin{cases} t_n^d = t_{trial}^d \\ \underline{e}_n = \underline{e}_{trial} \end{cases}$$

$$if \;\; f(\underline{t}_{trial}^{d}) > 0 \;\Rightarrow\; plastic\ step \;\; (\Delta\lambda \neq 0) \Rightarrow \begin{cases} f(\underline{t}^{d}) = 0 & \text{Yield condition} \\ \lambda \dot{f}(\underline{t}^{d}) = 0 & \text{Consistency condition} \end{cases}$$

$$\underline{e}_n^{dp} = \underline{e}_{n-1}^{dp} + \Delta\lambda \underline{t}_n^{d}$$

Dynamic relaxation method is used to calculate next step trial bond extension and bond force.

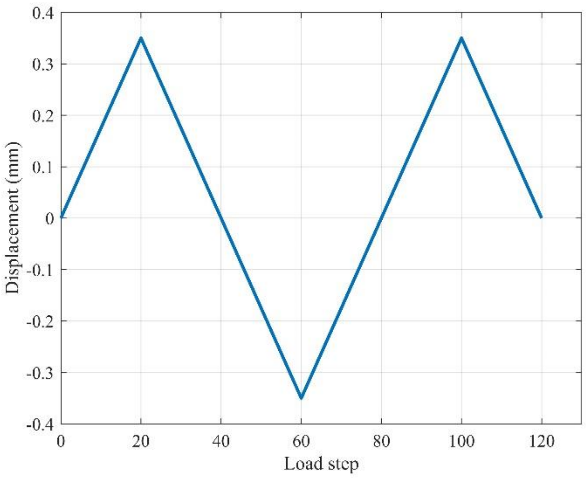
3D example with Hardening

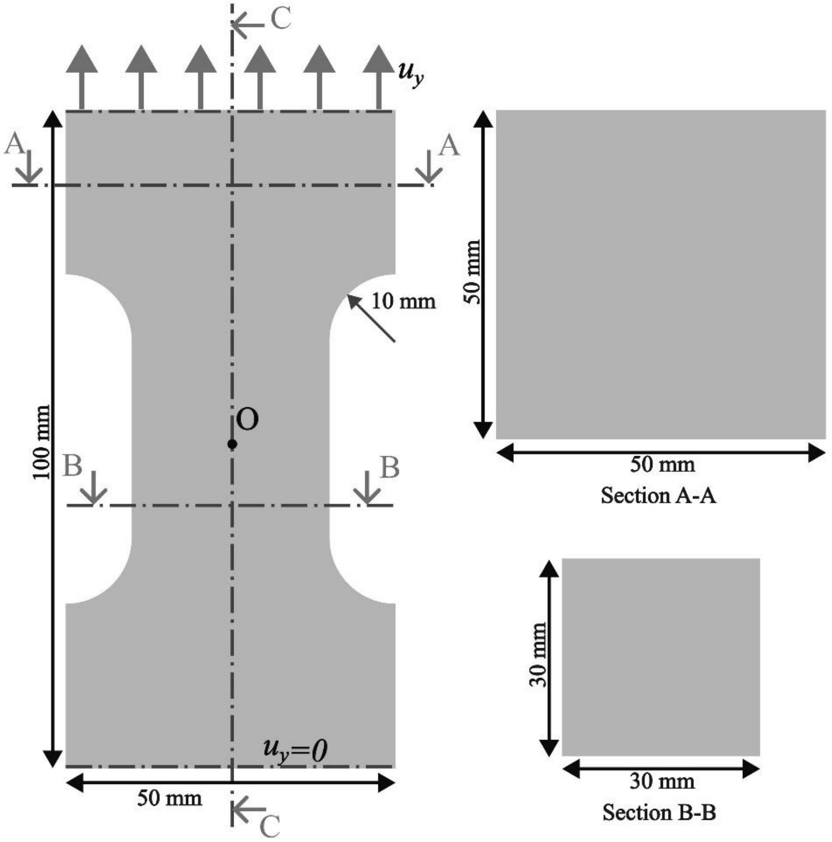
E=200 GPa υ=0.3

ρ=8000 kg/m^3

σ_y=600 MPa

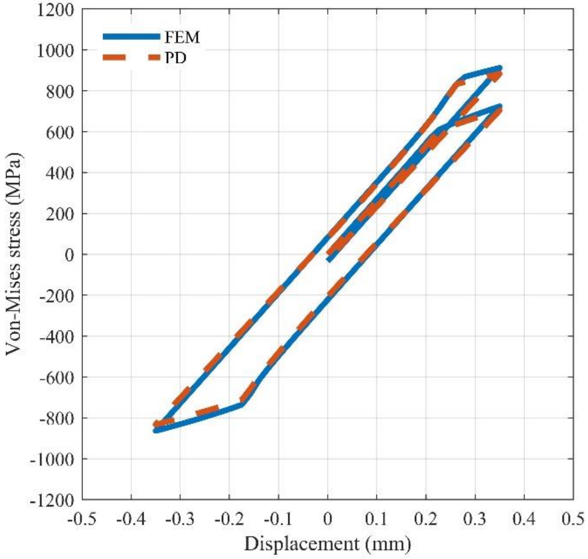
K=20 GPa H=20 GPa



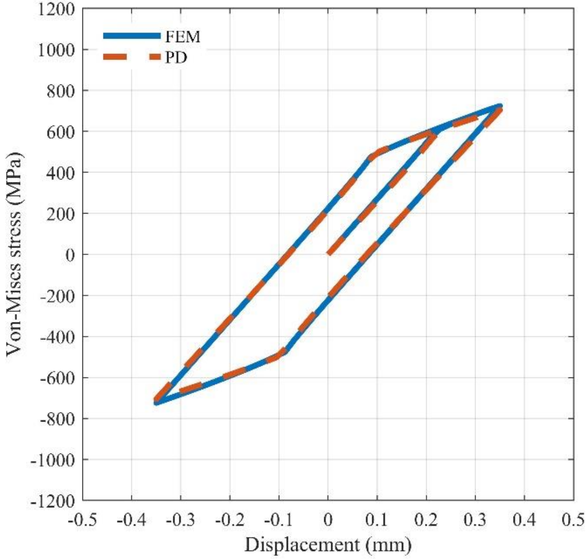


Elastoplastic Analysis of 3D example

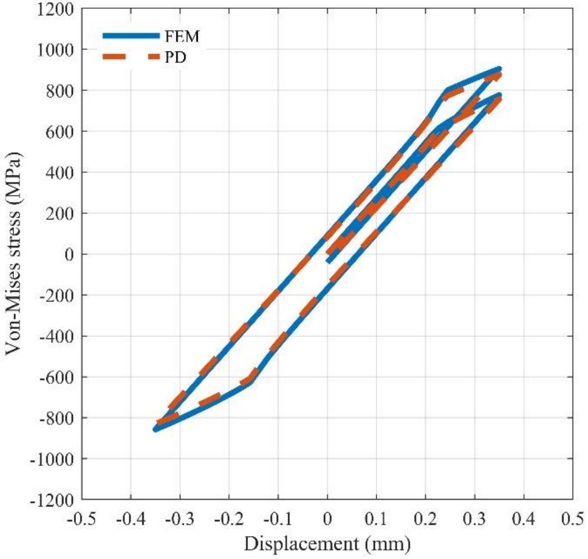
Stress at central node (Node O):



Isotropic hardening

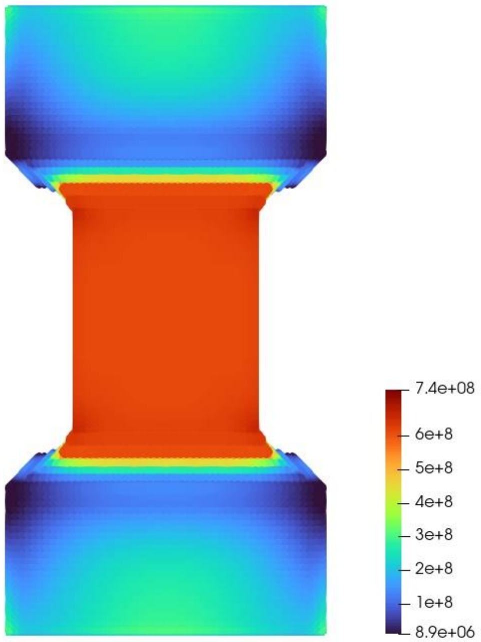


Kinematic hardening

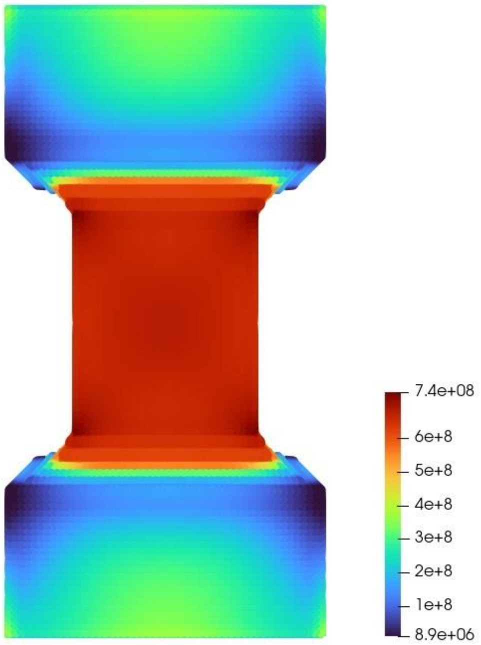


Mixed hardening

Elastoplastic Analysis of 3D example



Von-mises stress when u_y=0.21 mm



Von-mises stress when u_y=0.35 mm

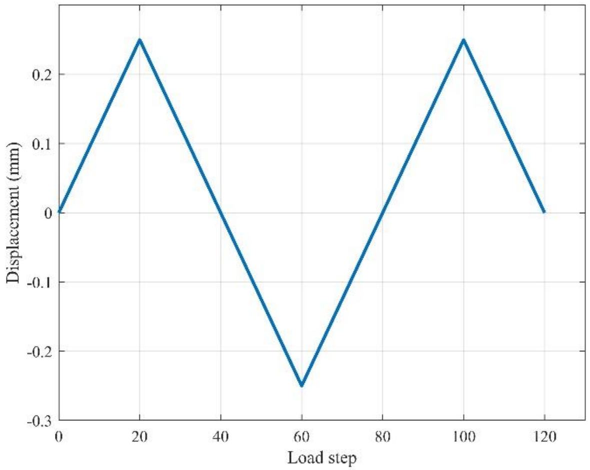
2D square plane stress example

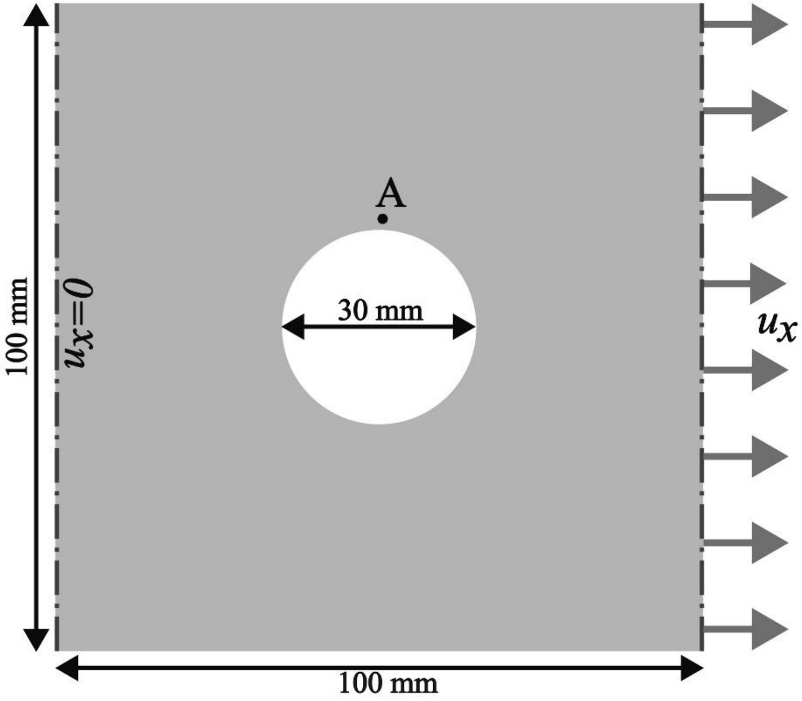
E=200 GPa υ=0.3

ρ=8000 kg/m³

σ_y=600 MPa

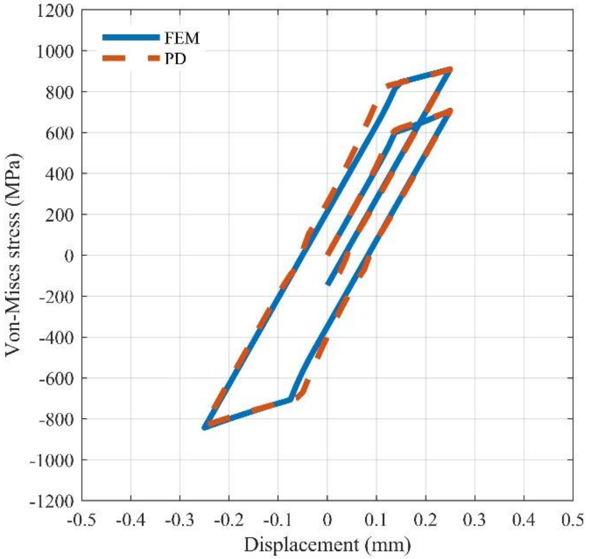
K=20 GPa H=20 GPa



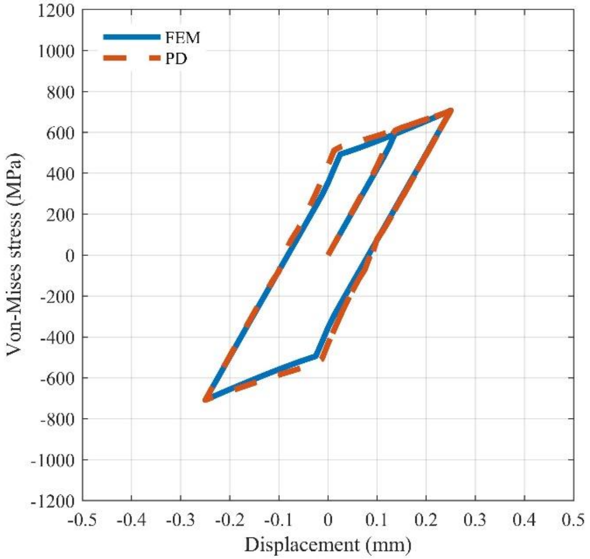


Elastoplastic Analysis of 2D example

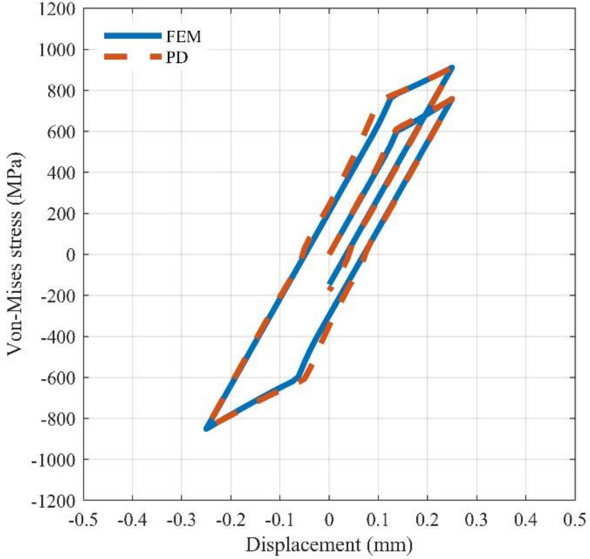
Stress at node A:



Isotropic hardening

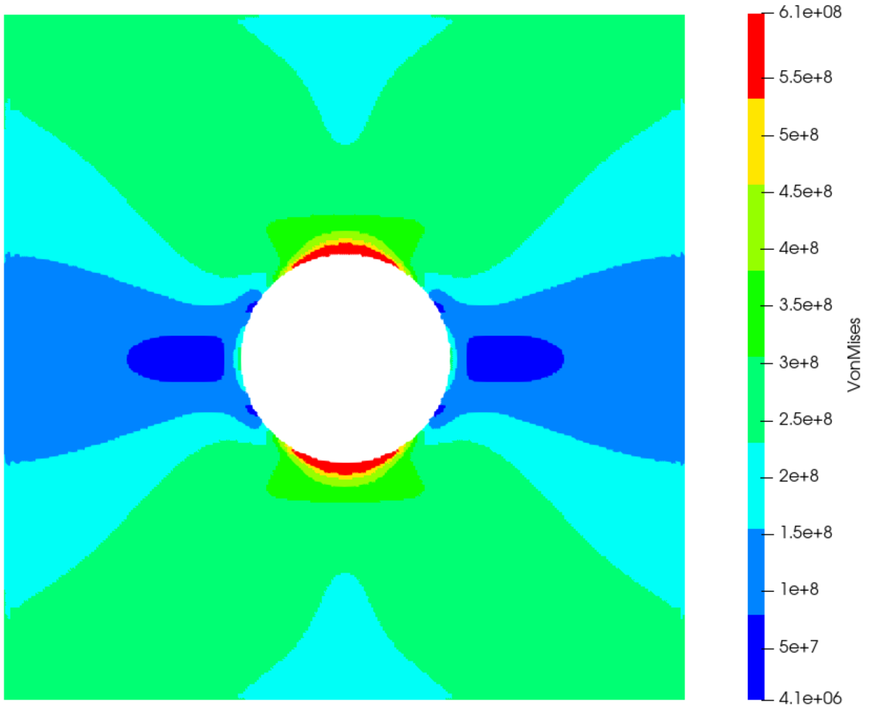


Kinematic hardening

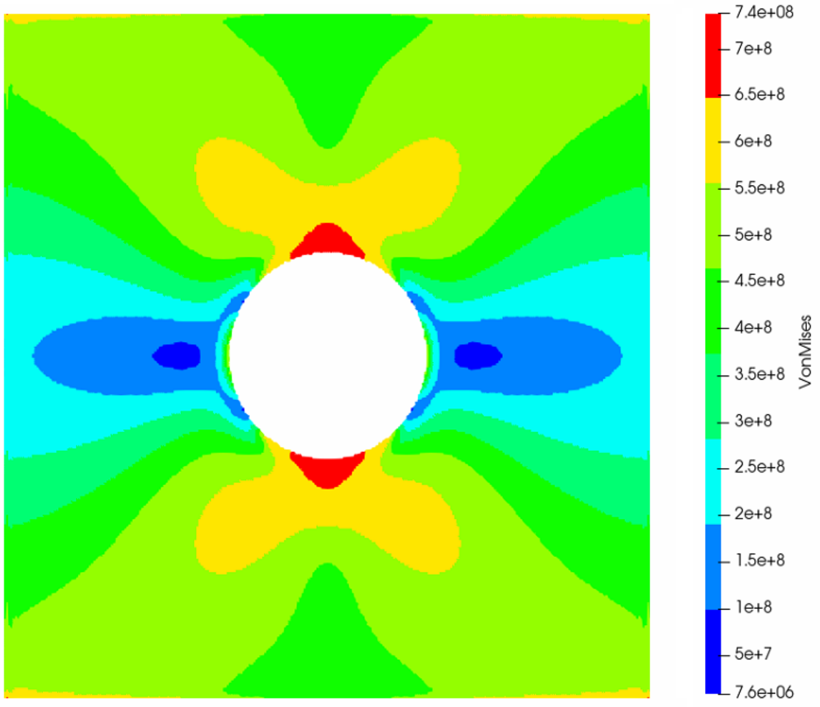


Mixed hardening

Elastoplastic Analysis of 2D example



Von-mises stress when u_x=0.125 mm



Von-mises stress when u_x=0.25 mm

Conclusion

- A new **elastoplastic Peridynamic** formulation was developed for analysis of **materials** with **isotropic** and **kinematic** hardening.
- **Ordinary state-based Peridynamic** model was used for the first time to consider the effects of shear deformation in plastic domain for 3D cases.
- In comparison with the methods used before, the proposed method is able to consider deformations with **large rotations** and accurate values of **strain hardening** for each step.
- Dynamic relaxation method was used to obtain quasi-static solutions of nonlinear Peridynamic equations.
- The elastoplastic Peridynamic approach will be used to study **crack propagation** in **elastoplastic materials**.

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Thanks for the attention

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