Double violation of Bell inequalities

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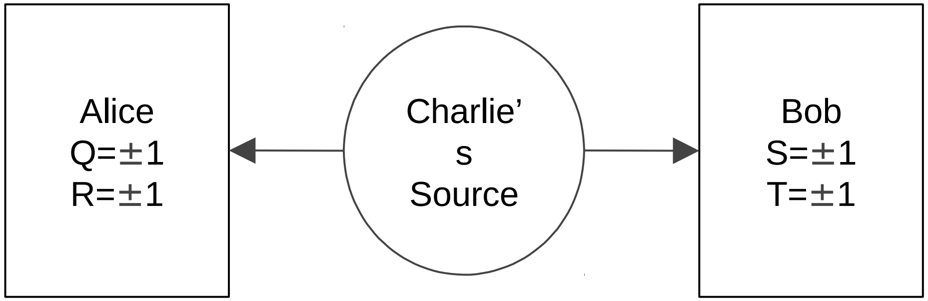
What is a Bell inequality?

Defines an experimental test to prove that Quantum Mechanics does not satisfy two fundamental principles, taken for granted in the classical theory:

> **locality:** two measurements, space-separeted and occouring at the same time, cannot influence each other.
>
> **realism:** physical systems (particles) possess *objective properties* that are independent from the measurements. Measurements merely reveal an objective property of the system.

Bell test: schematic representation



- Charlie has a source that prepares a couple of particles, which are sent to Alice and Bob.
- Alice (Bob) randomly selects the measurement: Q or R (S or T).
- Each measurement has two possible outcomes (+1 or -1).
- Alice and Bob’s <u>measurements are independent</u>. The timing of the experiment is arranged so that Alice and Bob do their measurements at the same time.

Bell inequality

Consider the quantity: $QS + RS + RT - QT = (Q + R)S + (R - Q)T$

Because R,Q = ±1 it follows that either (Q + R)S =0 or (R − Q)T =0, in either case QS + RS + RT - QT = ±2. Suppose next that, <u>before the measurements are performed</u>, p(q, r, s, t) is the probability that the system has *objective properties* so that Q = q, R = r, S = s, and T = t. These probabilities may depend on how Charlie prepares the two particles, and on experimental noise. Letting E(·) denote the mean value, we have

$$\mathbf{E}(QS + RS + RT - QT) = \sum_{qrst} p(q, r, s, t)(qs + rs + rt - qt) \leq 2$$

also

$$\mathbf{E}(QS + RS + RT - QT) = \mathbf{E}(QS) + \mathbf{E}(RS) + \mathbf{E}(RT) - \mathbf{E}(QT)$$

so we obtain the Bell inequality

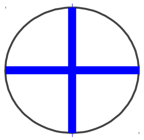
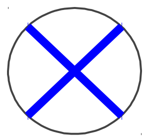
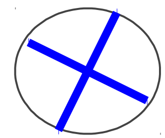
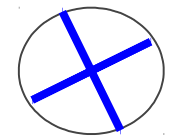
$$\mathbf{E}(QS) + \mathbf{E}(RS) + \mathbf{E}(RT) - \mathbf{E}(QT) \leq 2$$

Alice and Bob can estimate E(QS)+ E(RS)+ E(RT)−E(QT), by repeating the experiment many times.

Quantum mechanical prediction

Charlie prepares a quantum system of two entangled photons.

Alice and Bob perform measurements on the polarization of the photons:

Q  **R**  **S**  **T** 

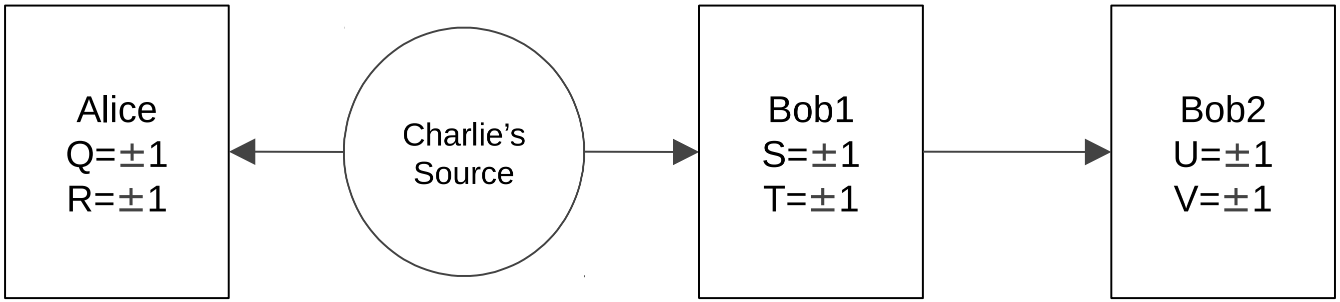
To each orientation of the polarization corresponds an outputs.

Calculating the average value QS+RS+RT-QT we have

$$\mathbf{E}(QS) + \mathbf{E}(RS) + \mathbf{E}(RT) - \mathbf{E}(QT) = 2\sqrt{2}$$

We have to reject at least one of the assumption we made: locality and realism.

Double Bell test



- Charlie prepares a couple of entagled photons.
- Alice and Bob2 performs a measurement on the polarization of the photon, randomly choosing two basis.
- Bob1 performs a *weak* measurement, recovering a partial information about the polarization of the photon.

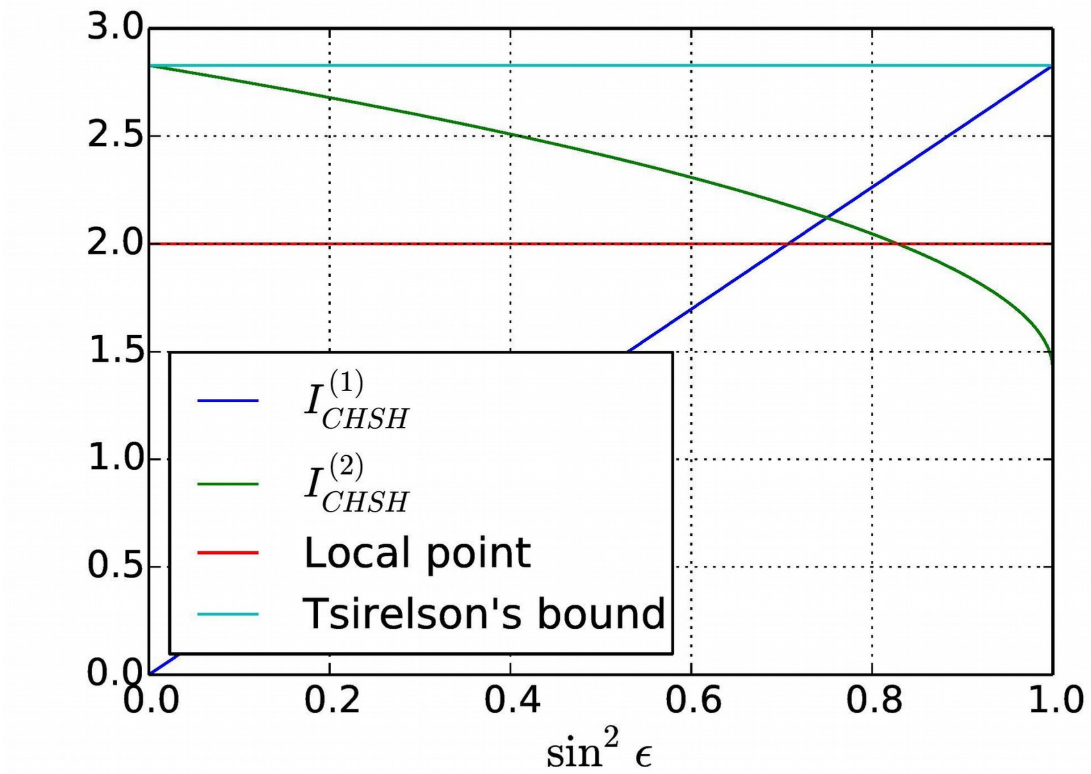
As an extention of the previous scenario, here we can consider two inequalities:

$$I^1 = E(QS)+E(RS)+E(RT)-E(QT) \leq 2 \qquad I^2 = E(QU)+E(RU)+E(RV)-E(QV) \leq 2$$

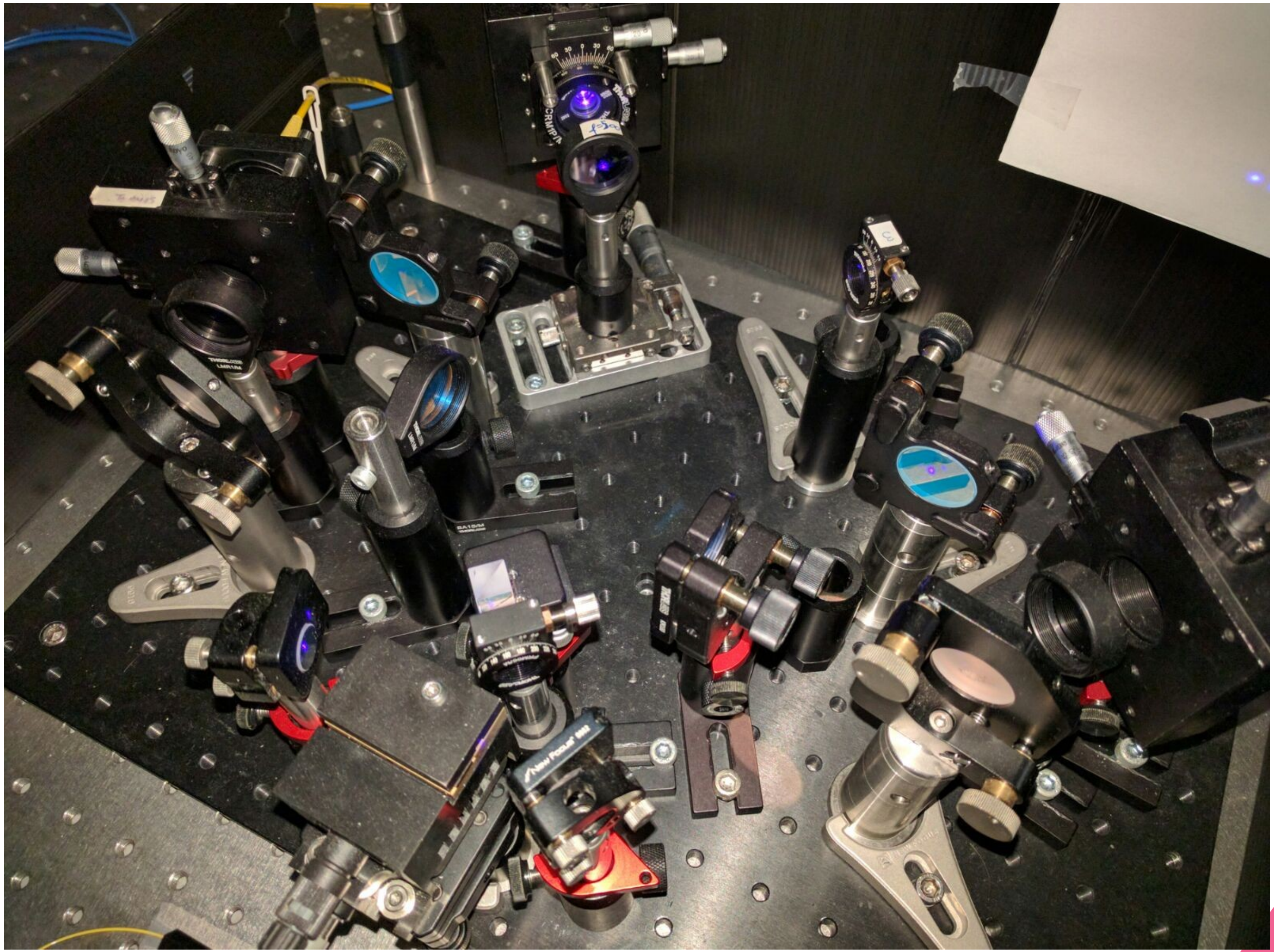
Are these violated by quantum mechanics? Consider that a measurement performed on a quantum state alters the state itself. Then Bob2 have access to a partial information about the state before Bob1's measurement.

Double violation of Bell inequalities

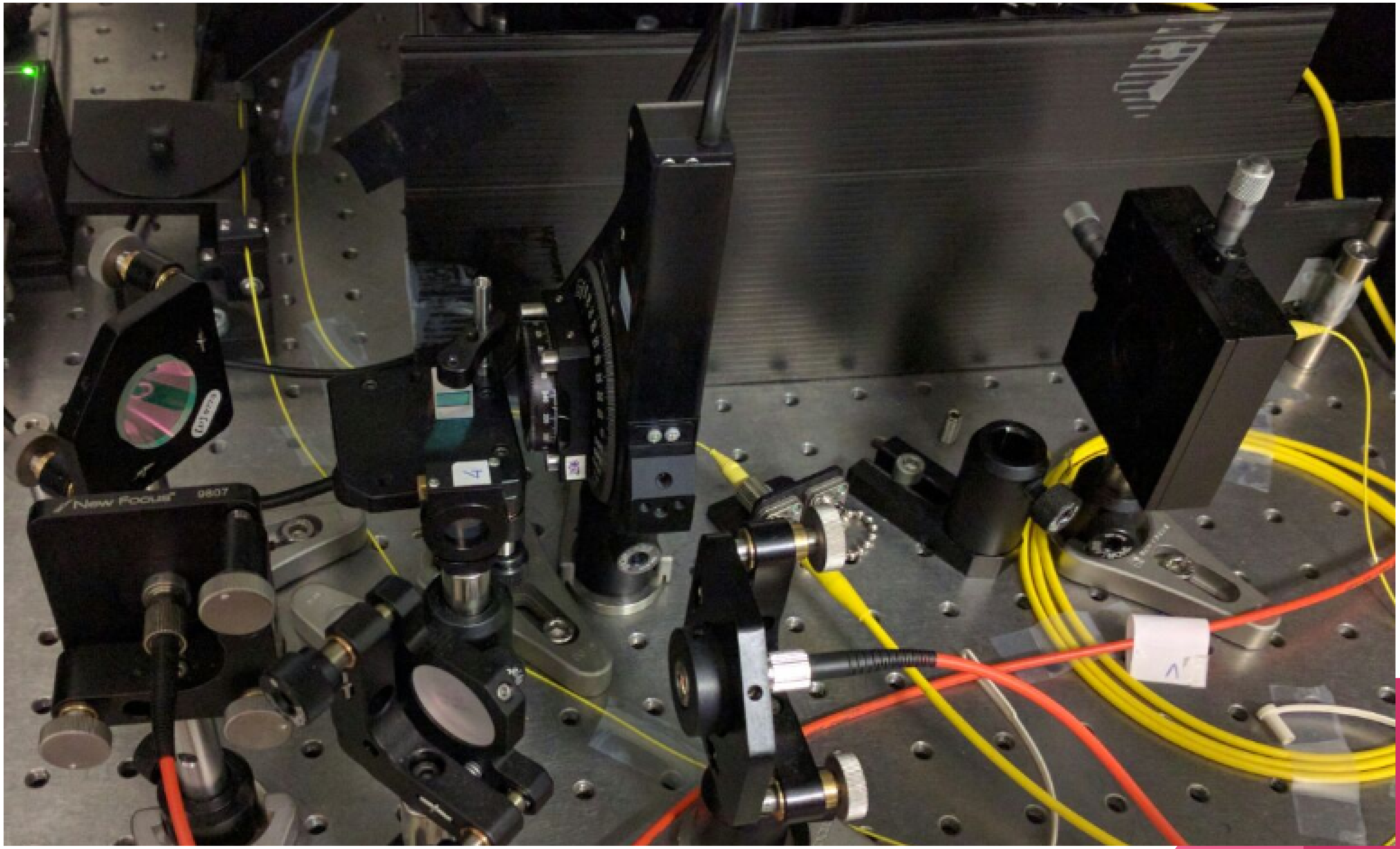
Quantum mechanics predicts the violation of both inequalities. There exist a range for the strenght of Bob1's measurements, in which this happens.



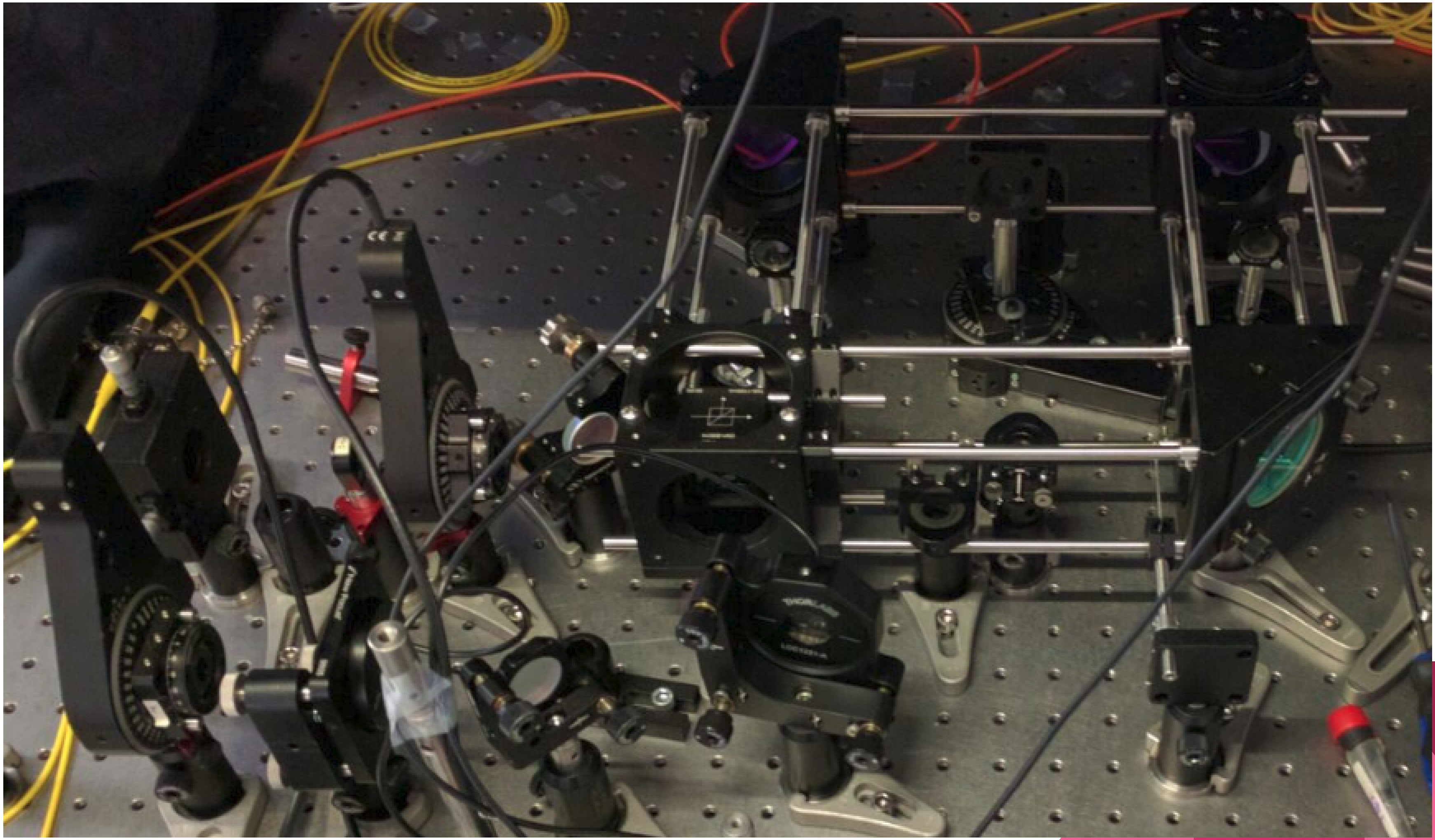
Experimental set up: Charlie



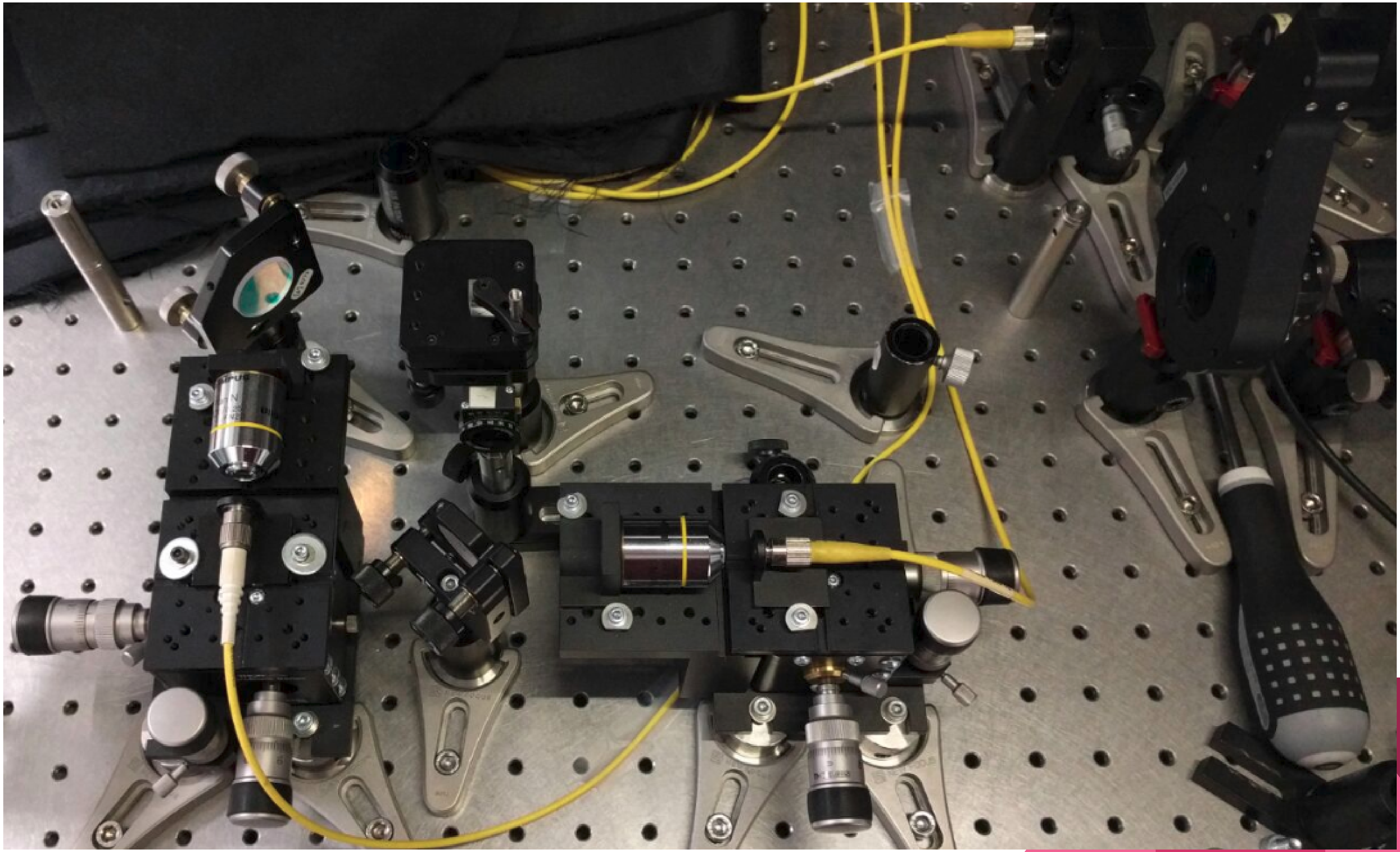
Experimental set up: Alice



Experimental set up: Bob1



Experimental set up: Bob2



Results

We have measured the violation of both inequalities.

More measurements are needed for a better estimation of the error.