Discontinuous mechanical problems studied with a peridynamics-based approach

Provided by: Soheil Bazazzadeh

Supervisor: Prof Ugo Galvanetto

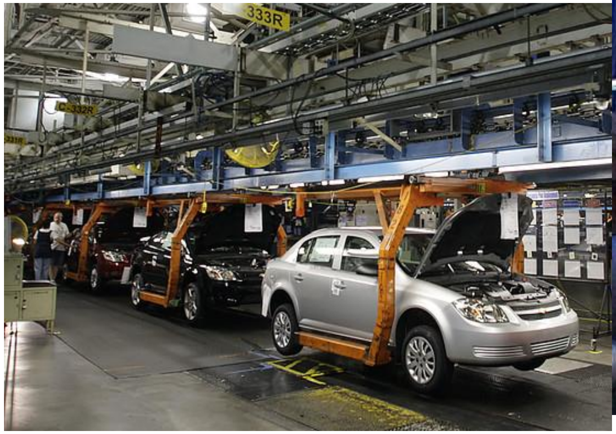
Advisor: Dr Mirco Zaccariotto

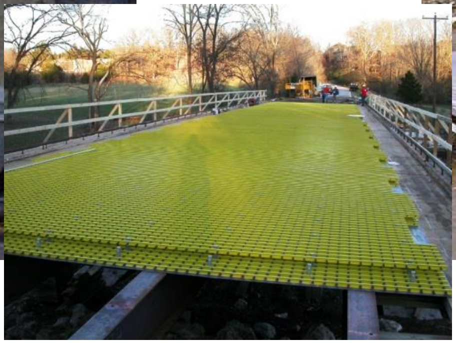
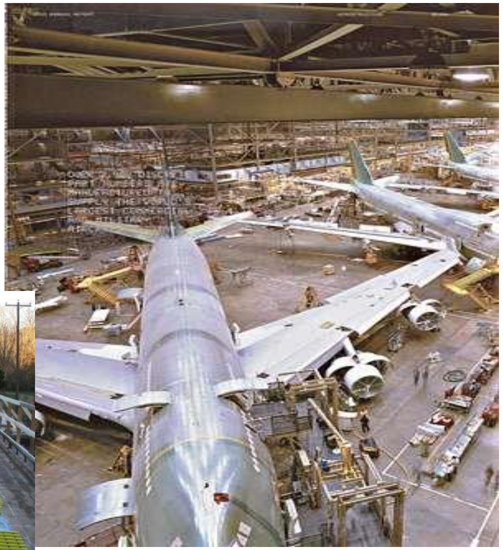
Index

- Aspects of modelling fatigue crack propagation
- Description of cylindrical model.
- Description of bilinear constitutive laws and three fatigue degradation strategies
- Numerical Results
- Peridynamics implementation of the fatigue model
- Future work
- Conclusions

Damage under high cycle fatigue

- Light weight structures with high resistance and stiffness
- Ocean, Maritime in coastal engineering
- Aerospace engineering
- Industrial engineering and transportation industry



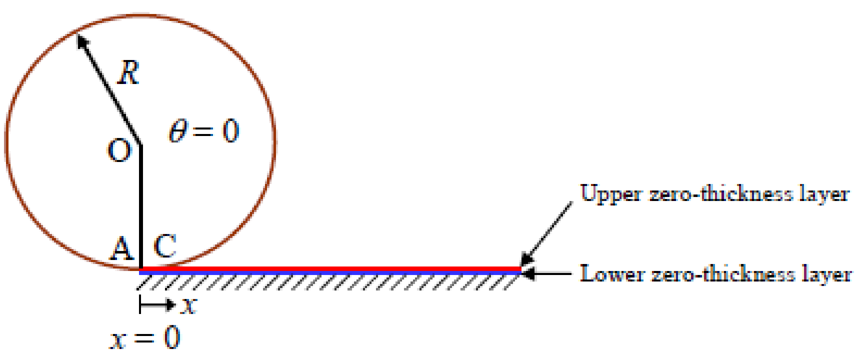




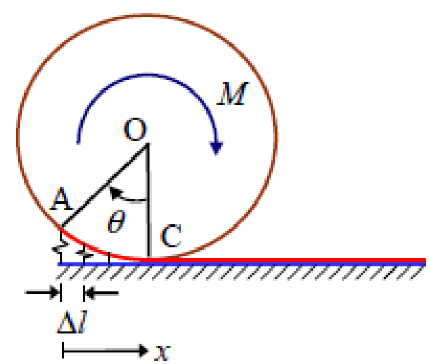
Cylinder model

- Consider a cylinder of radius *R* which, under static loading, rolls in a clockwise direction on a horizontal surface.

- The lower layer is fixed. The upper layer adheres to the surface of the cylinder at the contact point, *C* and remains adhered.

- As the cylinder rotates and the contact point advances as the cylinder rolls along the horizontal surface the angle swept through by OA is the rotation, θ, of the cylinder.



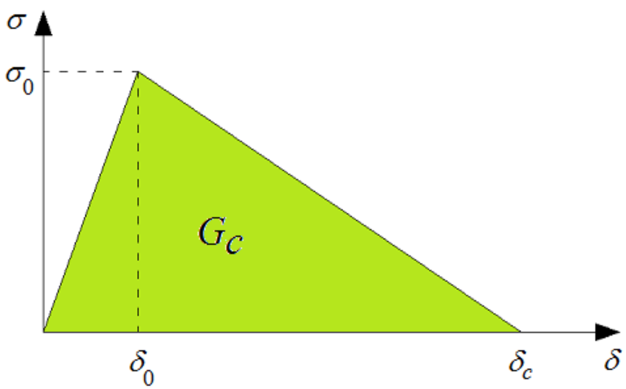


Constitutive laws

1- Bilinear constitutive law

K is the stiffness: $K = \sigma_0 / \delta_0$

Damage, D, is defined so that for the undamaged state it is equal to zero and for the failure state it is equal to unity



$$
\begin{cases}
\sigma = K\delta & 0 \le \delta \le \delta_0 & D = 0 \\
\sigma = \sigma_0 \left(\dfrac{\delta_c - \delta}{\delta_c - \delta_0} \right) & \delta_0 < \delta < \delta_c & 0 < D < 1 \\
\sigma = 0 & \delta_c < \delta & D = 1
\end{cases}
$$

The critical energy release rate: $G_c = \dfrac{\delta_c \sigma_0}{2}$

In fatigue problems two components of damage are considered

1-Static damage $\longrightarrow D_s$

2-Fatigue damage $\longrightarrow D_f$

Stiffness degradation based static damage

In this approach the static damage, D_S , can be considered a measure of the degradation of the initial stiffness

$$\sigma = (1 - D_s) K \delta$$

D_S can be written as

$$D_s = \begin{cases} 0 & 0 \le \delta \le \delta_0 \\ \left(\dfrac{\delta - \delta_0}{\delta} \right) \left(\dfrac{\delta_c}{\delta_c - \delta_0} \right) & \delta_0 < \delta < \delta_c \\ 1 & \delta \ge \delta_c \end{cases}$$

Static and Fatigue degradation strategies

- Rate of change of the static damage

$$\frac{\partial D_s}{\partial t} = \frac{\delta_0 \delta_c}{\delta_c - \delta_0} \frac{\dot{\delta}}{\delta^2} \qquad \delta_0 < \delta < \delta_c$$

$$D_{s\,N+\Delta N} - D_{s\,N} = \frac{\delta_0 \delta_c}{\delta_c - \delta_0}\left(\frac{1}{\delta_N} - \frac{1}{\delta_{N+\Delta N}}\right)$$

- **Fatigue** damage Strategies
  - First strategy
  - Second strategy
  - Third strategy

First strategy

Fatigue rate is

$$\dot{D}_f = \frac{\partial D_f}{\partial N} = Ce^{\lambda D}\left(\frac{\delta}{\delta_c}\right)^{\beta}$$

Increment of fatigue damage

$$D_{f\,N+\Delta N} - D_{f\,N} = \Delta N \frac{C}{1+\beta} e^{\lambda D_\mu} \left(\frac{\delta_\mu}{\delta_c}\right)^{1+\beta}$$

The increment of total damage due to an increment of the number of cycles (*ΔN*) is

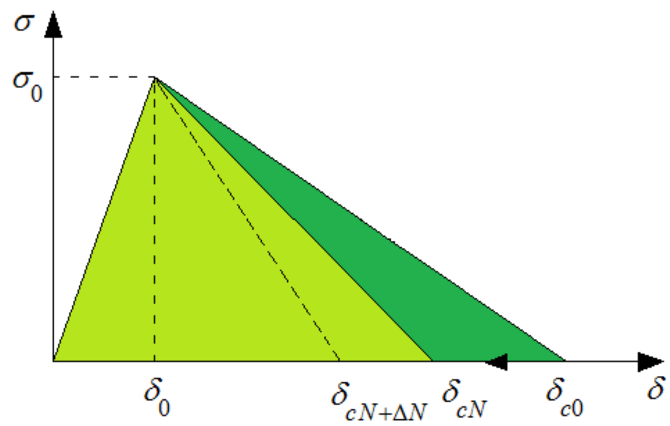
$$D_{N+\Delta N} - D_N = \underbrace{\frac{\delta_0 \delta_c}{\delta_c - \delta_0}\left(\frac{1}{\delta_N} - \frac{1}{\delta_{N+\Delta N}}\right)}_{static} + \underbrace{\Delta N \frac{C}{1+\beta} e^{\lambda D_\mu}\left(\frac{\delta_\mu}{\delta_c}\right)^{1+\beta}}_{fatigue}$$

$$D_\mu = (1-\mu)D_N + \mu D_{N+\Delta N}$$

$$\delta_\mu = (1-\mu)\delta_N + \mu \delta_{N+\Delta N}$$

Second strategy

Failure elongation of a spring in each increment of load cycle, ΔN , is a function of displacement as follows



$$\delta_{c\,N+\Delta N} = \delta_{c\,N} - B\,(\delta_{N+\Delta N} - \delta_0)^n\,\Delta N$$

$\delta_{c\,N+\Delta N}$: Is the failure displacement of a spring after $N + \Delta N$ load cycles

B, n : Constants related to the material behaviour

The total damage at the end of each load cycle increment can be obtained as

$$D_{N+\Delta N} = \frac{\delta_0 \delta_{c\,N+\Delta N}}{\delta_{c\,N+\Delta N} - \delta_0}\left(\frac{1}{\delta_0} - \frac{1}{\delta_{N+\Delta N}}\right)$$

Accordingly, the total damage increment in this approach is given by

$$D_{N+\Delta N} - D_N = \frac{\delta_0 \delta_{c\,N+\Delta N}}{\delta_{c\,N+\Delta N} - \delta_0}\left(\frac{1}{\delta_0} - \frac{1}{\delta_{N+\Delta N}}\right) - \frac{\delta_0 \delta_{c\,N}}{\delta_{c\,N} - \delta_0}\left(\frac{1}{\delta_0} - \frac{1}{\delta_N}\right)$$

In the above formula if $\Delta N \to 0$

$$D_{N+\Delta N} - D_N = \frac{\delta_0 \delta_{c\,N}}{\delta_{c\,N} - \delta_0}\left(\frac{1}{\delta_0} - \frac{1}{\delta_{N+\Delta N}}\right) - \frac{\delta_0 \delta_{c\,N}}{\delta_{c\,N} - \delta_0}\left(\frac{1}{\delta_0} - \frac{1}{\delta_N}\right) = \underbrace{\frac{\delta_0 \delta_{c\,N}}{\delta_{c\,N} - \delta_0}\left(\frac{1}{\delta_N} - \frac{1}{\delta_{N+\Delta N}}\right)}_{\text{Static Damage}}$$

Third Fatigue strategy

It is assumed that the fatigue damage term has an expression as

$$\dot{D}_f = \frac{\partial D_f}{\partial N} = A' \left(\frac{\delta}{\delta_c} \right)^{m'}$$

$$\int_{N}^{N+\Delta N} \frac{\partial D_f}{\partial N} dN \approx A' \Delta N \left(\frac{1}{2} \left(\frac{\delta_N}{\delta_c} + \frac{\delta_{N+\Delta N}}{\delta_c} \right) \right)^{m'}$$

A' and m' are constants related to the material behaviour

$\delta_{N+\Delta N}$: displacement of each spring after $N + \Delta N$ load cycles

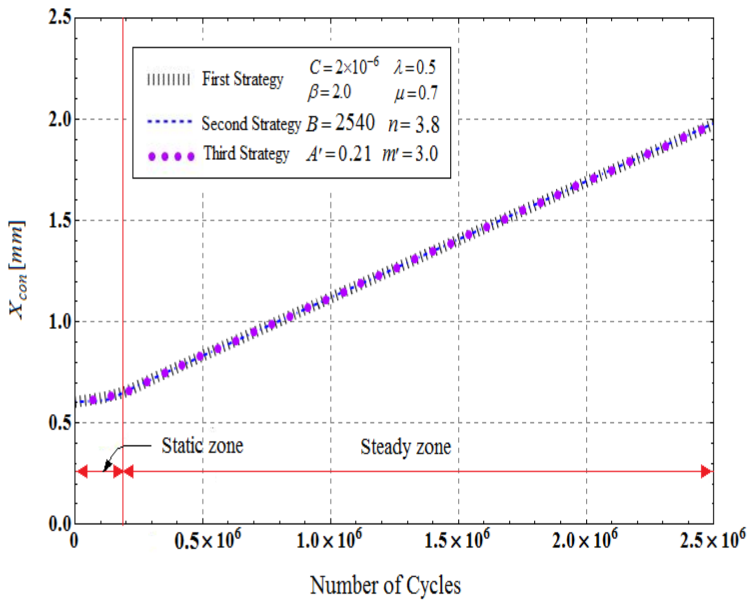
The increment of total damage due to an increment of the number of cycles (*ΔN*) is

$$D_{N+\Delta N} - D_N = \underbrace{\frac{\delta_0 \delta_c}{\delta_c - \delta_0} \left(\frac{1}{\delta_N} - \frac{1}{\delta_{N+\Delta N}} \right)}_{static} + \underbrace{A' \Delta N \left(\frac{1}{2} \left(\frac{\delta_N}{\delta_c} + \frac{\delta_{N+\Delta N}}{\delta_c} \right) \right)^{m'}}_{fatigue}$$

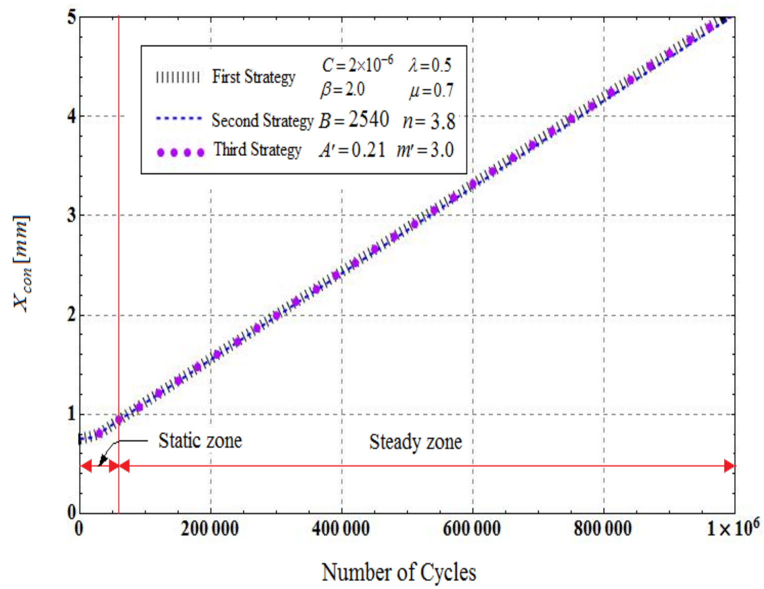
Examples of fatigue behavior simulation

$R = 100 \text{ mm} \quad Gc = 0.26 \text{ J/m}^2 \quad \delta_c = 0.017 \text{ mm}$

$\Delta l = 0.005 \text{ mm} \quad \Delta N = 100$



$$M_a = 0.2M_c$$



$$M_a = 0.3M_c$$

Total amount of time

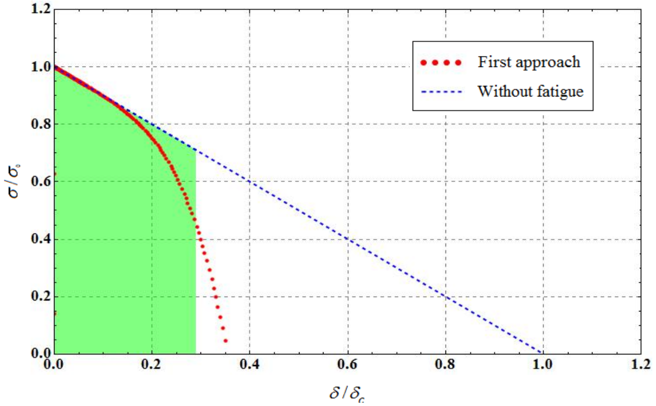
This Problem has been solved by an Intel(R) Core(TM) i7-2600K CPU @ 3.40GHz machine and the computational time reported are based on the CPUsec of this system

First Approach : 67.92 CPUs

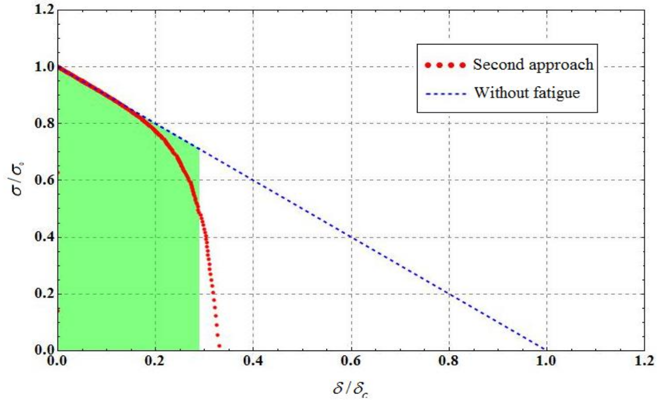
Second Approach: 59.45 CPUs

Third Approach: 57.00 CPUs

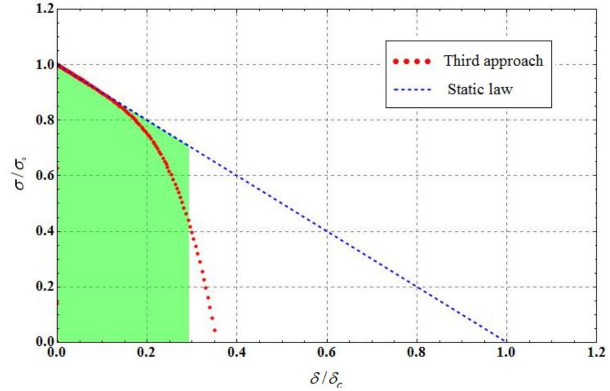
Single spring behavior under static and fatigue loading using three proposed laws



First approach

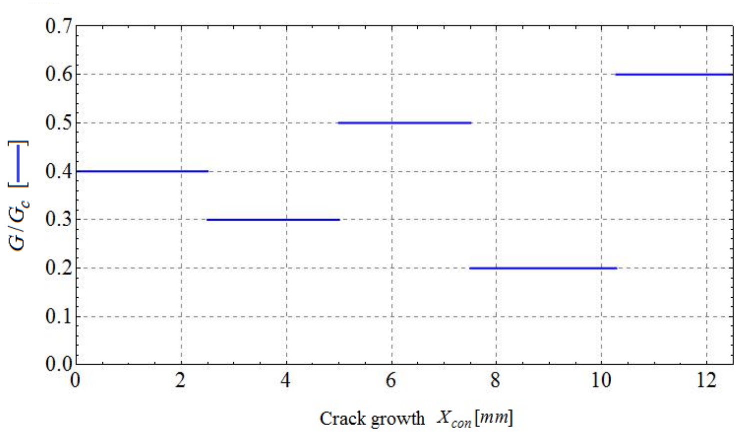


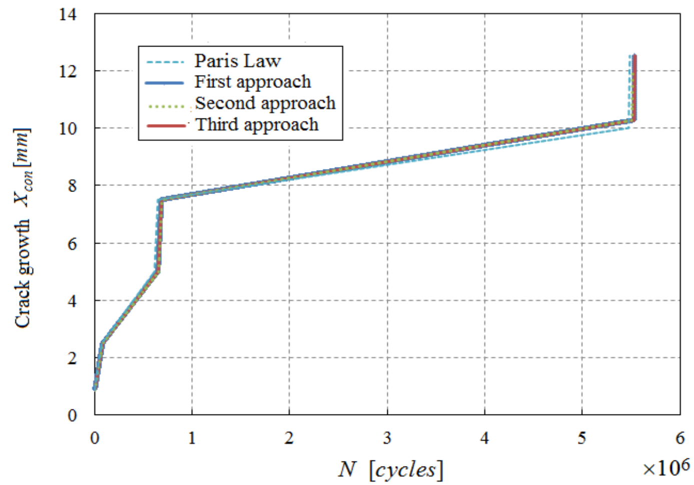
Second approach



Third approach

We compare the performance of the three fatigue degradation strategies when used with a variable G (i.e a M_a in the case of the cylinder model).



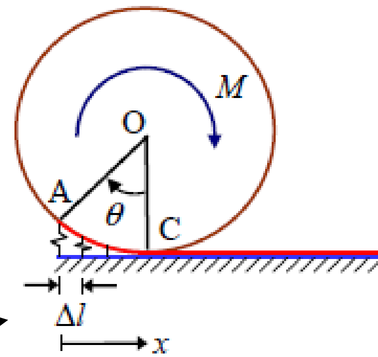


The three fatigue degradation strategies seem to have a very similar behaviour:

1) Provide the same slope *Xcon* vs *N*

2) Provide similar curves σ/σ_0 vs δ/δ_c

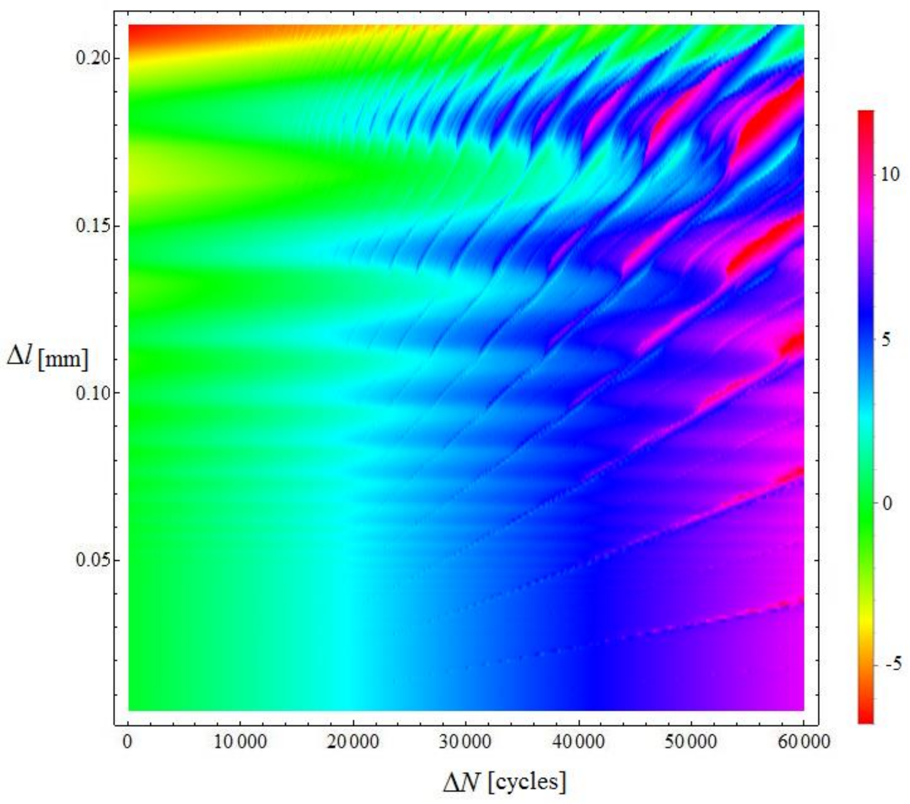
3) Provide similar curves *Xcon* vs *N* with variable *G*



Let us now compare how they perform by varying *Δl* and *ΔN*.

- ‘**Exact slope**’ (dX_{con}/dN) computed numerically using a small value of Δl (=0.005) and ΔN (=100).
- The value of the ‘exact slope’ is the same for all three formulations.

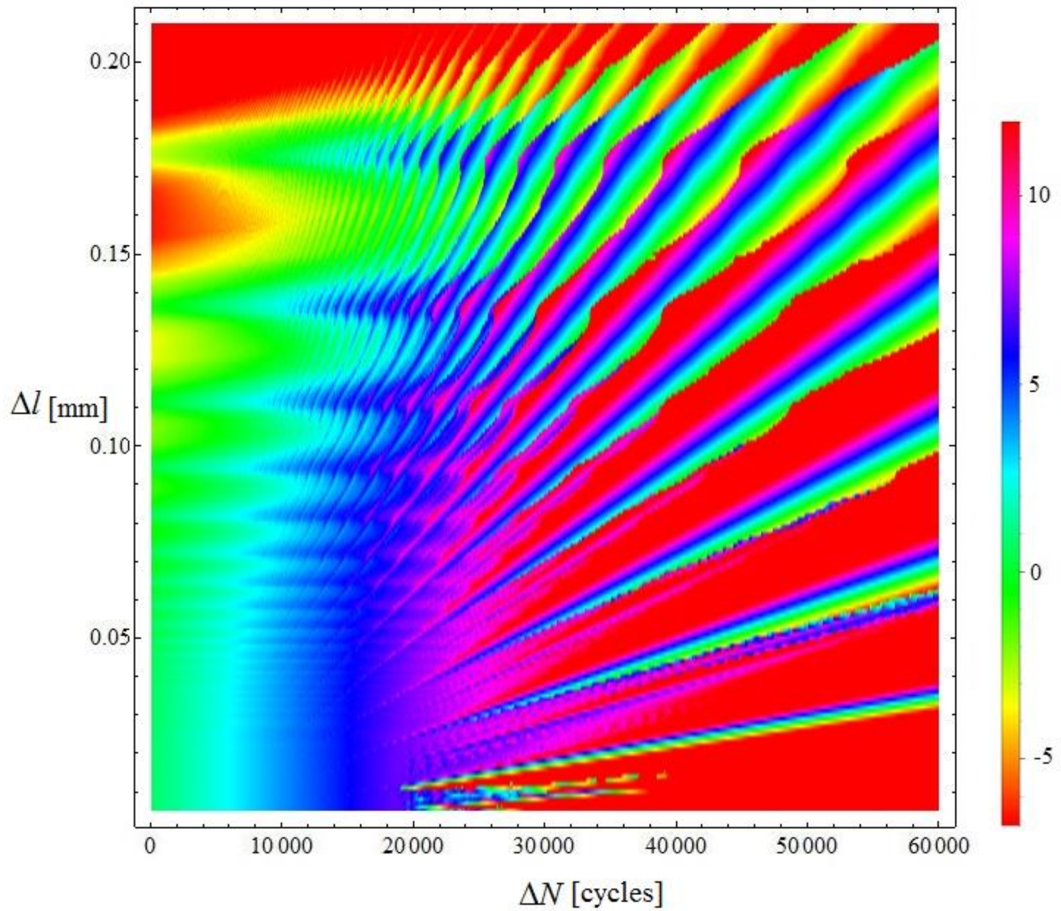
Error plots for first fatigue Law



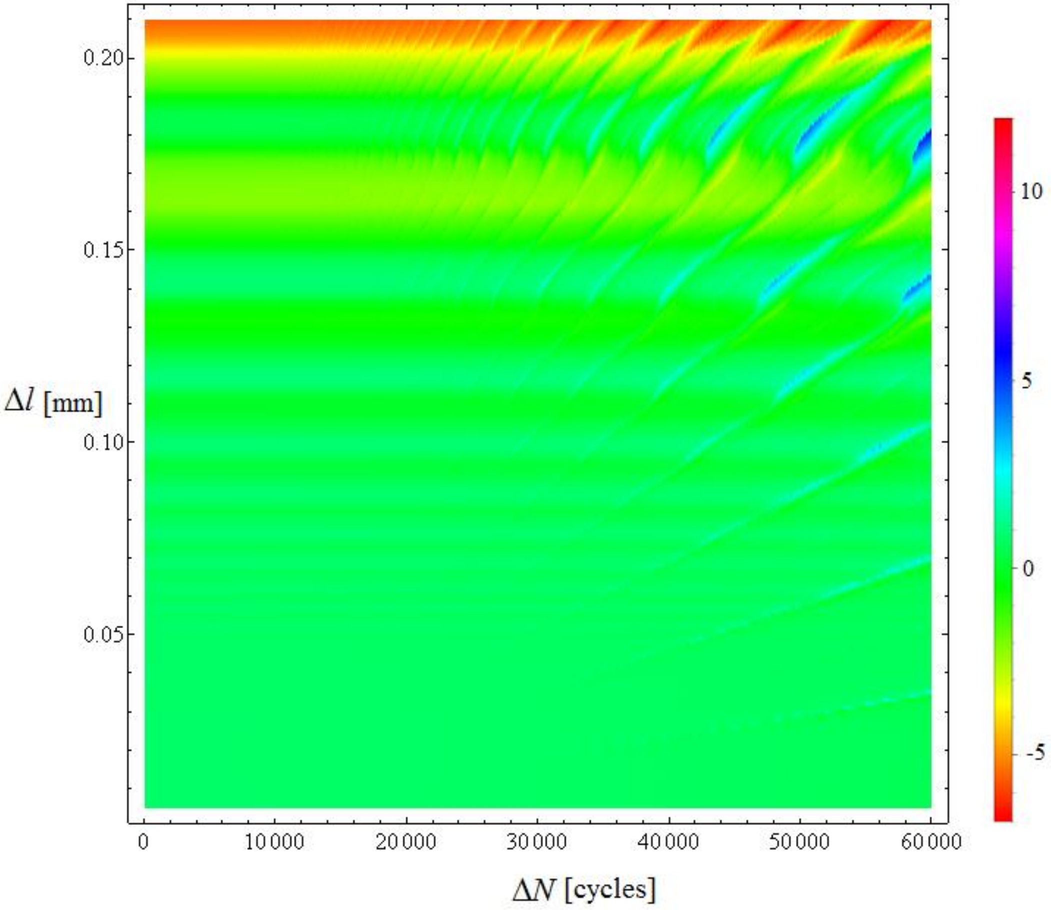
- The range of Δl values has been divided in 210 uniform subintervals and the range of ΔN values in 600 uniform subintervals so that 123600 different parameter combinations have been considered for each case.

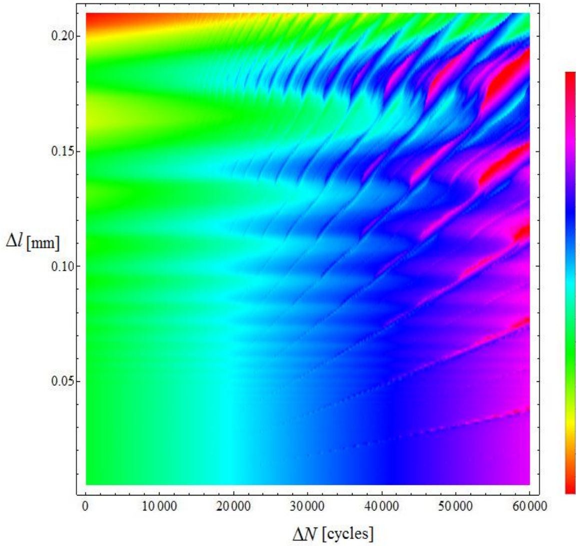
$$0.005 < \Delta l < 0.21$$
$$100 < \Delta N < 60000$$

Error plots for second fatigue Law

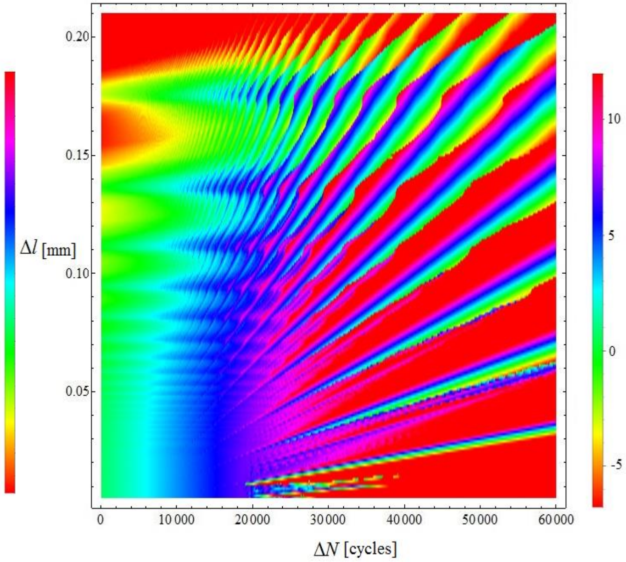


Error plots for third fatigue Law

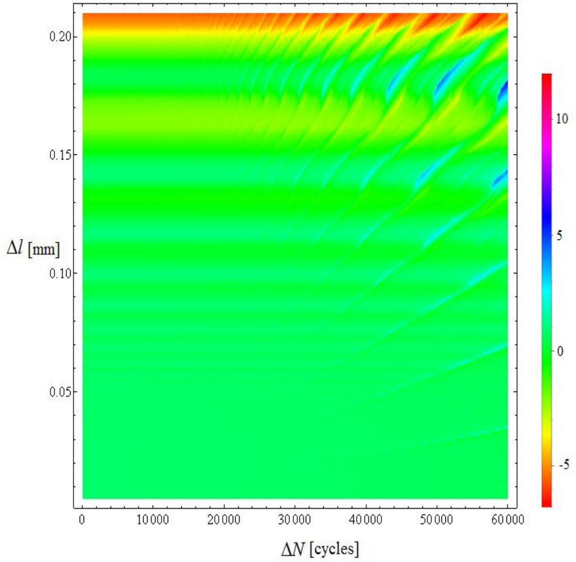




First fatigue Law



Second fatigue Law



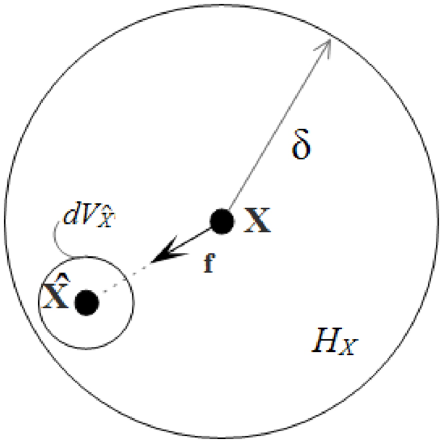
Third fatigue Law

Therefore, the simple cylinder model seems to suggest that the third fatigue law is preferable

Peridynamics implementation of the fatigue model

The peridynamic static equilibrium equation is written as

$$\mathbf{0} = \int_{H_x} \mathbf{f}(\mathbf{u}(\hat{\mathbf{x}},t) - \mathbf{u}(\mathbf{x},t), \hat{\mathbf{x}} - \mathbf{x}) \mathrm{dV}_{\hat{\mathbf{x}}} + \mathbf{b}(\mathbf{x},t) \quad \textit{for} \quad \mathbf{x} \in \Omega \textit{ and } t \in [t_0, t_f]$$



The stretch of the bond can be expressed as

$$s = \frac{|\boldsymbol{\xi} + \boldsymbol{\eta}| - |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|} \qquad \boldsymbol{\xi} = \hat{\mathbf{x}} - \mathbf{x}, \quad \boldsymbol{\eta} = \mathbf{u}(\hat{\mathbf{x}},t) - \mathbf{u}(\mathbf{x},t)$$

The pairwise force function $\mathbf{f}$ and the stretch s are related as

$$\mathbf{f} = c\, s\, \mu(\boldsymbol{\xi}) \frac{\boldsymbol{\eta} + \boldsymbol{\xi}}{|\boldsymbol{\eta} + \boldsymbol{\xi}|} \cong c\, s\, \mu(\boldsymbol{\xi}) \frac{\boldsymbol{\xi}}{|\boldsymbol{\xi}|}$$

$$\mu(\boldsymbol{\xi}) = \begin{cases} 1 \longrightarrow \text{Active bonds} \\ 0 \longrightarrow \text{Broken bonds} \end{cases}$$

s

The micro-modulus c in terms of the Young's modulus E

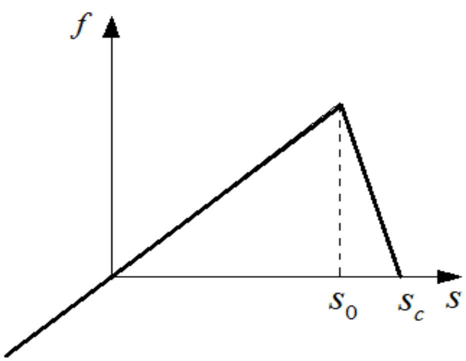
and of the horizon radius δ

$$c = \frac{12E}{\pi\delta^4} \quad \text{3D case}$$

$$\begin{cases} c = \dfrac{9E}{\pi t_h \delta^3} & \text{Plane stress} \\ c = \dfrac{48E}{5\pi t_h \delta^3} & \text{Plane strain} \end{cases}$$

Nonlinearity of the model

Magnitude of the bond force f vs bond stretch s for Elastic-progressively damaging material



$$s_0 = \sqrt{\frac{5G_0}{6E\delta}} \quad \text{3D case}$$

$$\begin{cases} s_0 = \sqrt{\dfrac{4\pi G_0}{9E\delta}} & \text{Plane stress} \\ s_0 = \sqrt{\dfrac{5\pi G_0}{12E\delta}} & \text{Plane strain} \end{cases}$$

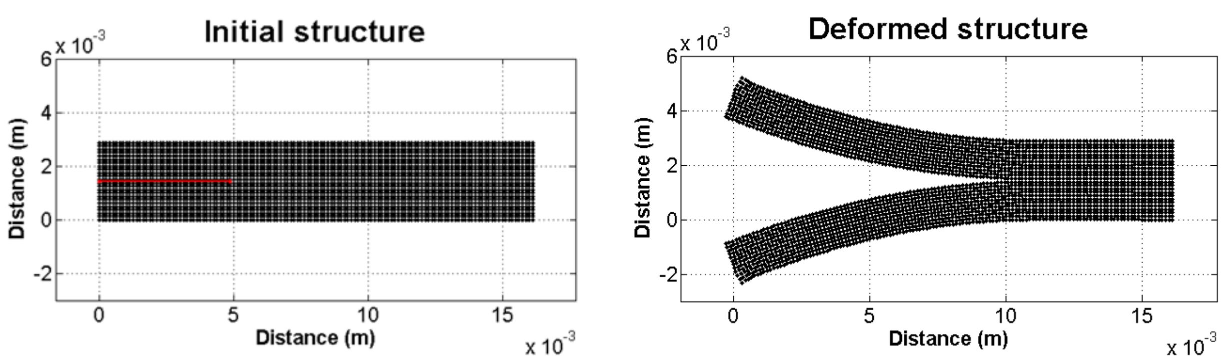
G_0 is the energy required to start the damaging process.

$$k_r = \frac{G_c}{G_0} = \frac{s_c}{s_0}$$

Using a bilinear constitutive law, one may define the pairwise force function magnitude

$$\begin{cases} f(s) = c\,s & 0 \le s \le s_0 \\ f(s) = c\,(1-D)\,s & s_0 < s < s_c \\ f(s) = 0 & s \ge s_c \end{cases}$$

Double cantilever Beam Peridynamic Example



$L = 0.0161$ m

$W = 0.0029$ m

$E = 70$ GPa

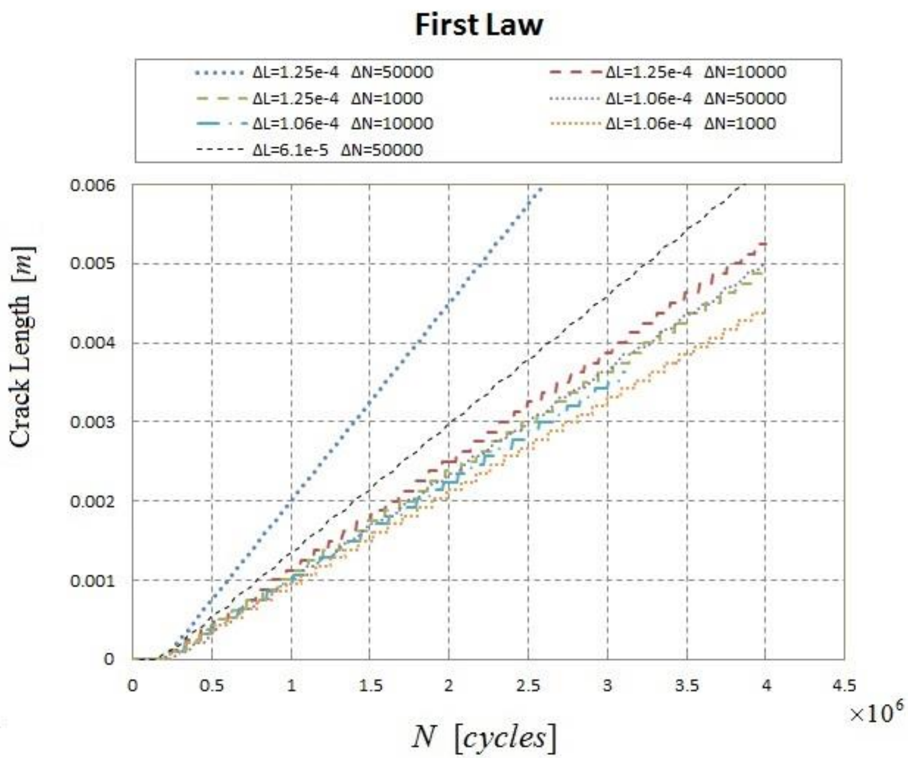
$a = 0.005$ m

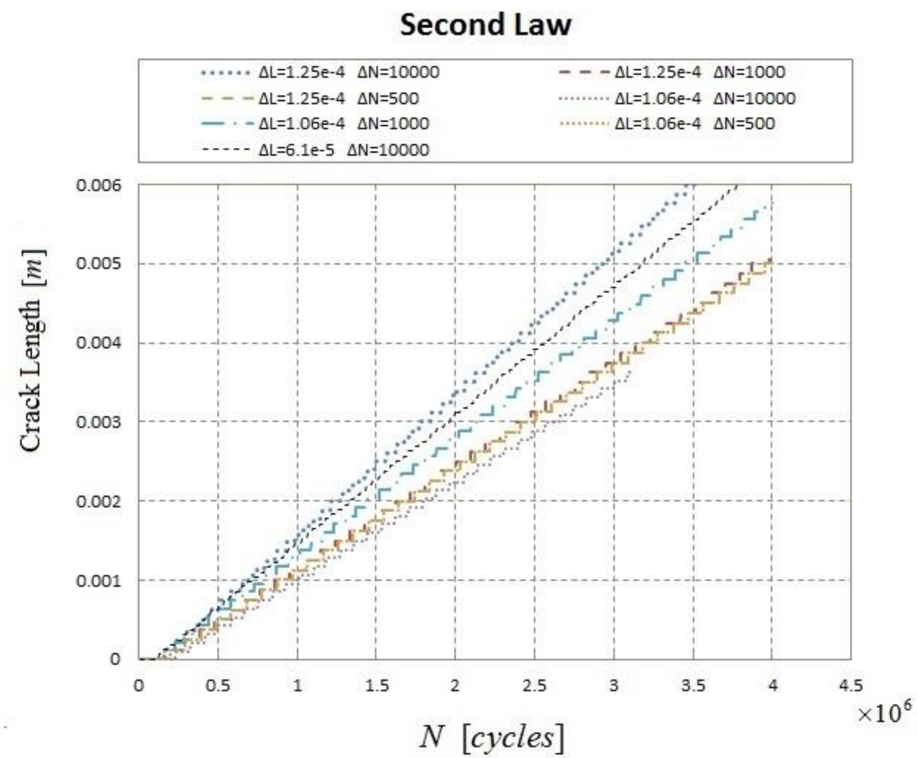
$k_r = 20$

$M_a = 0.048$ Nm

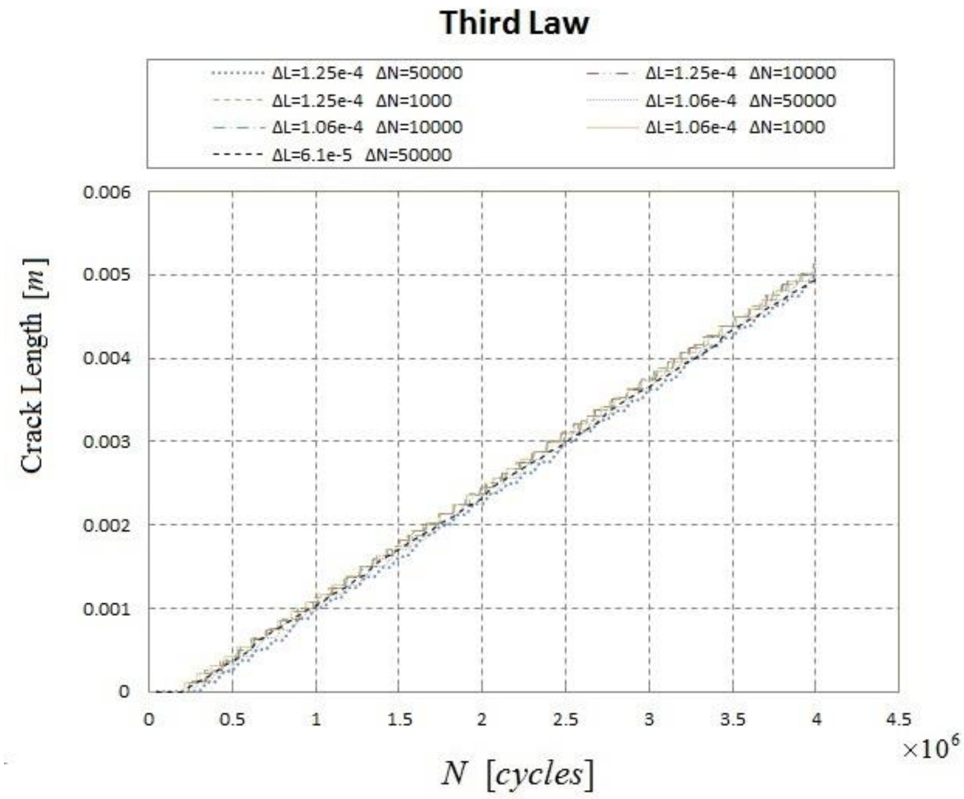
$$\text{grid spacing} \begin{cases} \Delta l = 6.27 \times 10^{-5}\ \text{m} \\ \Delta l = 1.06 \times 10^{-4}\ \text{m} \\ \Delta l = 1.25 \times 10^{-4}\ \text{m} \end{cases} \quad \Delta N = \begin{cases} 500 \\ 1000 \\ 10000 \\ 50000 \end{cases}$$

In this example, the crack length with respect to the number of load cycles, N, is computed by using the three fatigue strategies





- Both the cylinder model and the DCB discretized with the peridynamic based code confirm that the third law is the best of the three considered in the present work because it is more stable with respect to the variations of the discretization parameters and it is cheaper from a computational point of view.



Conclusions

- We compare three fatigue degradation strategies to be used in the simulation of fatigue crack propagation.

- A one degree of freedom cylinder model is used to carry out an efficient comparison of the computational performance of the three fatigue degradation strategies. Then the three laws are implemented in a code using bond based peridynamics to simulate fatigue crack propagation

- The third fatigue degradation strategy is the best, among those investigated, for being used in BBPD codes.

- The cylinder model appears to provide reliable indications about the computational performance of the fatigue laws.

Thanks For Your Attention