DISCONTINUOUS MECHANICAL PROBLEMS STUDIED WITH A PERIDYNAMICS-BASED APPROACH

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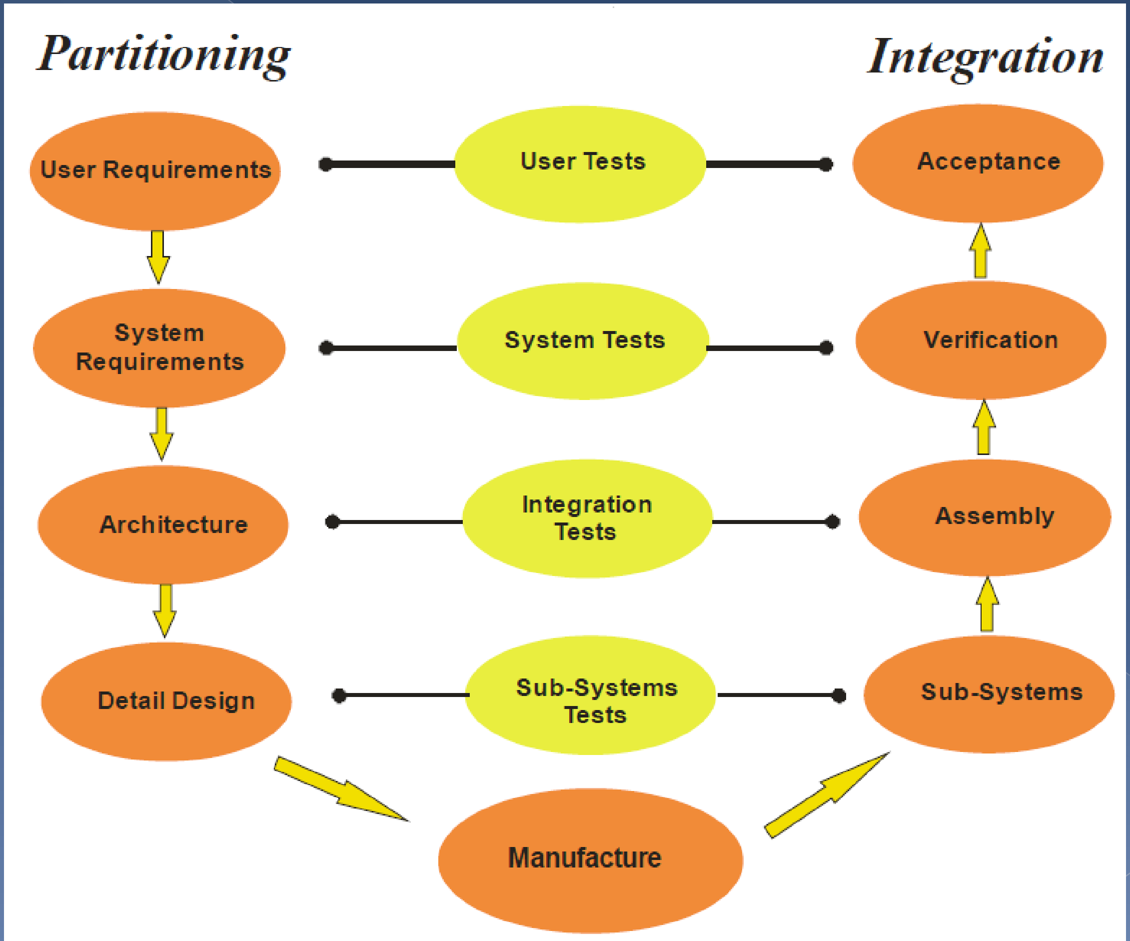
Fracture Mechanics

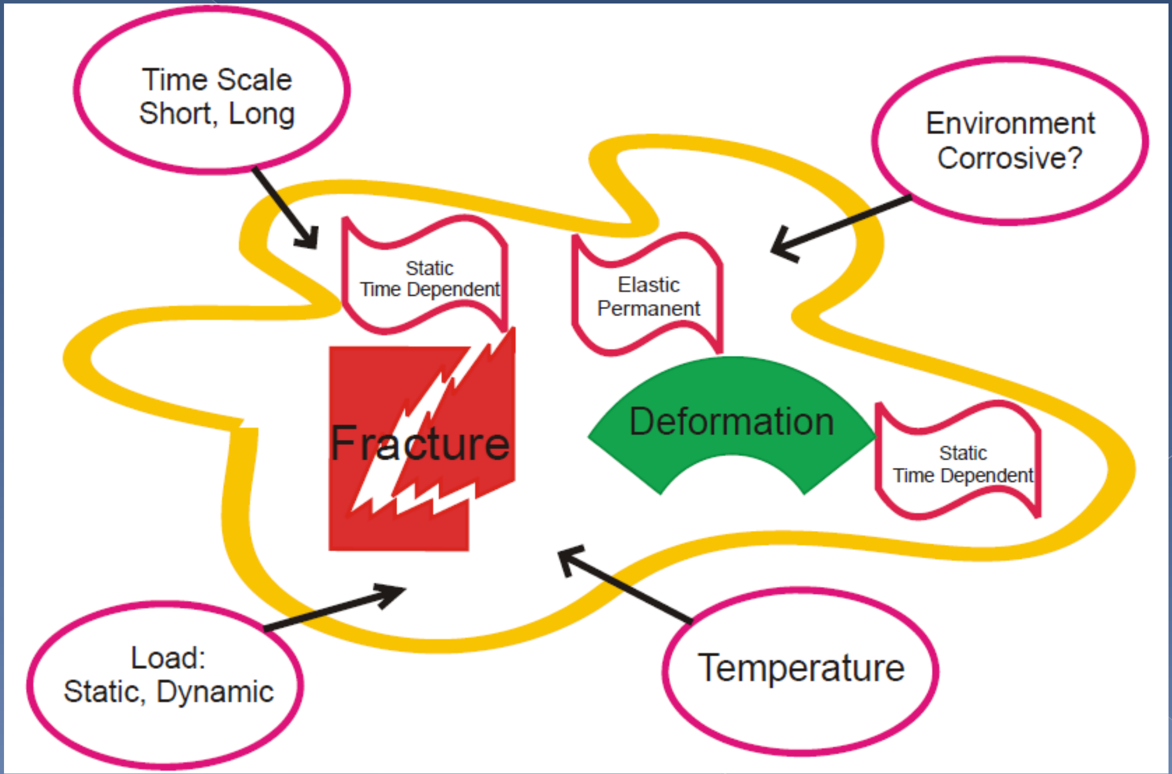
Some Fundamental Aspects of Structural Design and Failure Analysis

The ultimate goal: **To be able to design structures or components that are capable of safely withstanding static or dynamic service loads for a certain period of time.**



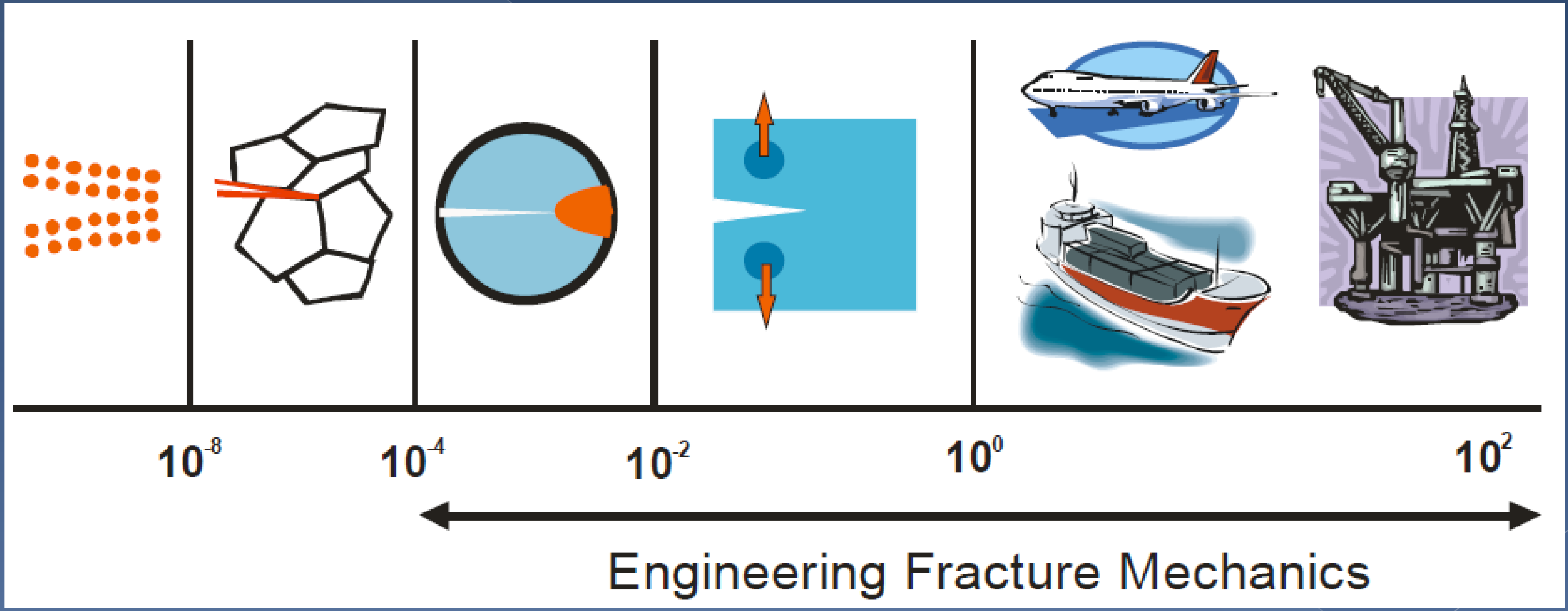
In general, various failure mechanisms may be classified into the two broad fields of Deformation and Fracture.
A more detailed list is:

- Large Elastic Deformation
- Unstable Elastic Deformation (Buckling)
- Impact & fragmentation (dynamics crack)
- Coupled problems (cracks propagation due to stress-corrosion and environmental effect)
- Stress Corrosion Cracking
- Plastic Deformation
- Fracture
- Fatigue
- Creep



Fracture Mechanics

Fracture mechanics is a field of solid mechanics that deals with the mechanical behavior of cracked bodies.



Historically, the first major step in the direction of quantification of the effects of cracklike defects was taken by C. E. Inglis (a professor of Naval Architecture).

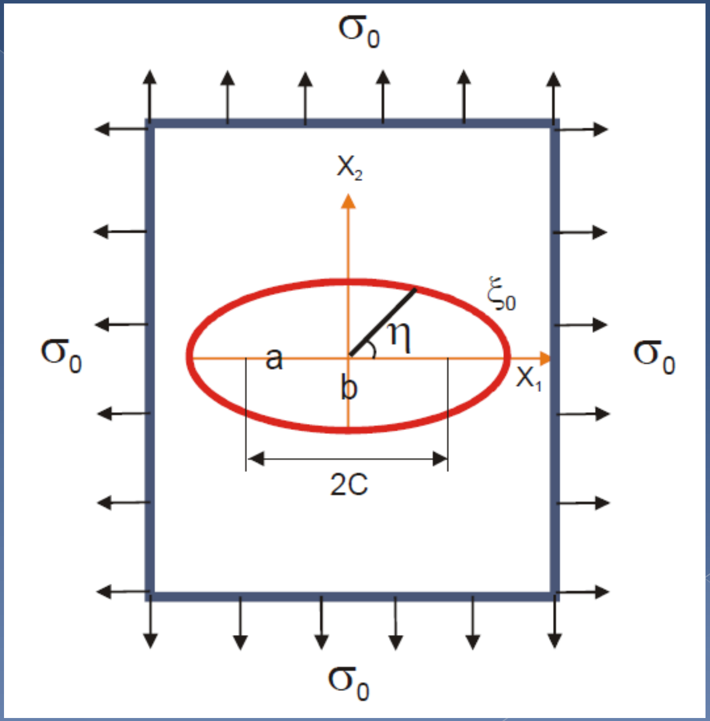
1913 → **Stress analysis for an elliptical hole in an infinite linear elastic plate loaded at its outer boundaries**

making the minor axis very much less than the major, a cracklike discontinuity can be modeled

Min and Max Stress

$$\left(\sigma_\eta\right)_{\eta=0,\pi}^{\max} = 2\sigma_0 \frac{a}{b}$$

$$\left(\sigma_\eta\right)_{\eta=\frac{\pi}{2},\frac{3\pi}{2}}^{\min} = 2\sigma_0 \frac{b}{a}$$



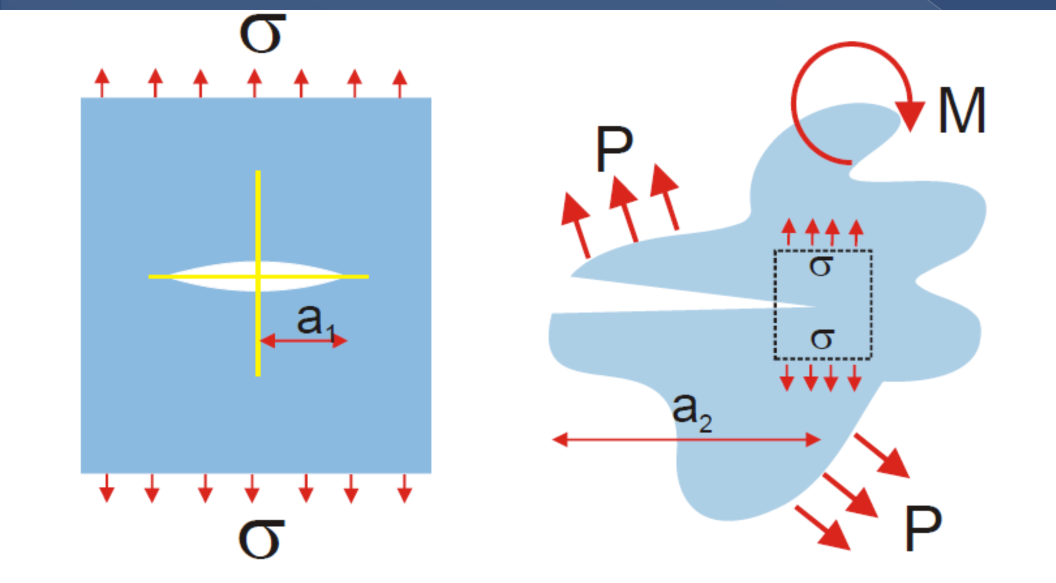
In 1956 → Irwin by the aid of Griffith theory developed the energy release rate concept

$$G = -\frac{d\Pi}{dA} \geq R$$

→ Useful for engineering problems

Westergaard approach → stresses and displacements near the crack tip could be described by a single parameter

→ ***Stress intensity factor***



$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = K_o \frac{\cos \theta/2}{\sqrt{2\pi r}} \begin{bmatrix} 1 - \sin \theta/2 \sin 3\theta/2 \\ 1 + \sin \theta/2 \sin 3\theta/2 \\ \sin \theta/2 \cos 3\theta/2 \end{bmatrix} + \text{negligible higher order terms}; \quad K_o = \sigma \sqrt{\pi a}$$

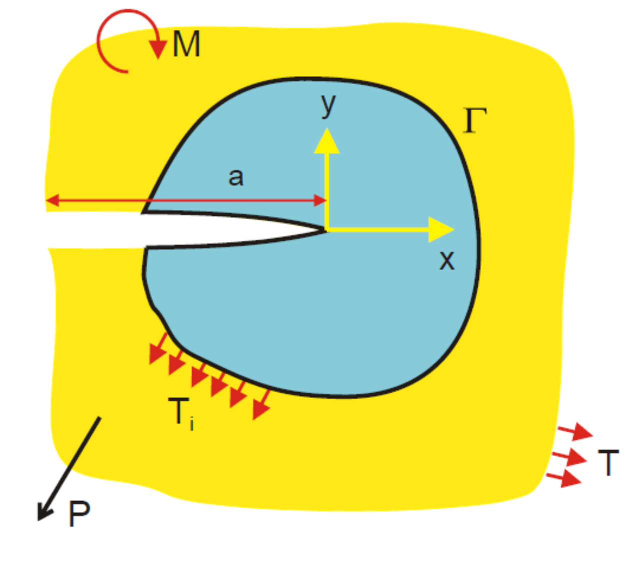
Elasto-Plastic Fracture Mechanics

- In the regime where the global stress-strain response of the body is linear and elastic (LEFM), the elastic energy release rate, G, and the stress intensity factor K can be used for characterizing cracks in structures
- In the elastic-plastic region (EPFM) also called yielding fracture mechanics (YFM), the fracture characterizing parameters are the J-integral and the crack-tip-opening displacement, CTOD.

J contour integral

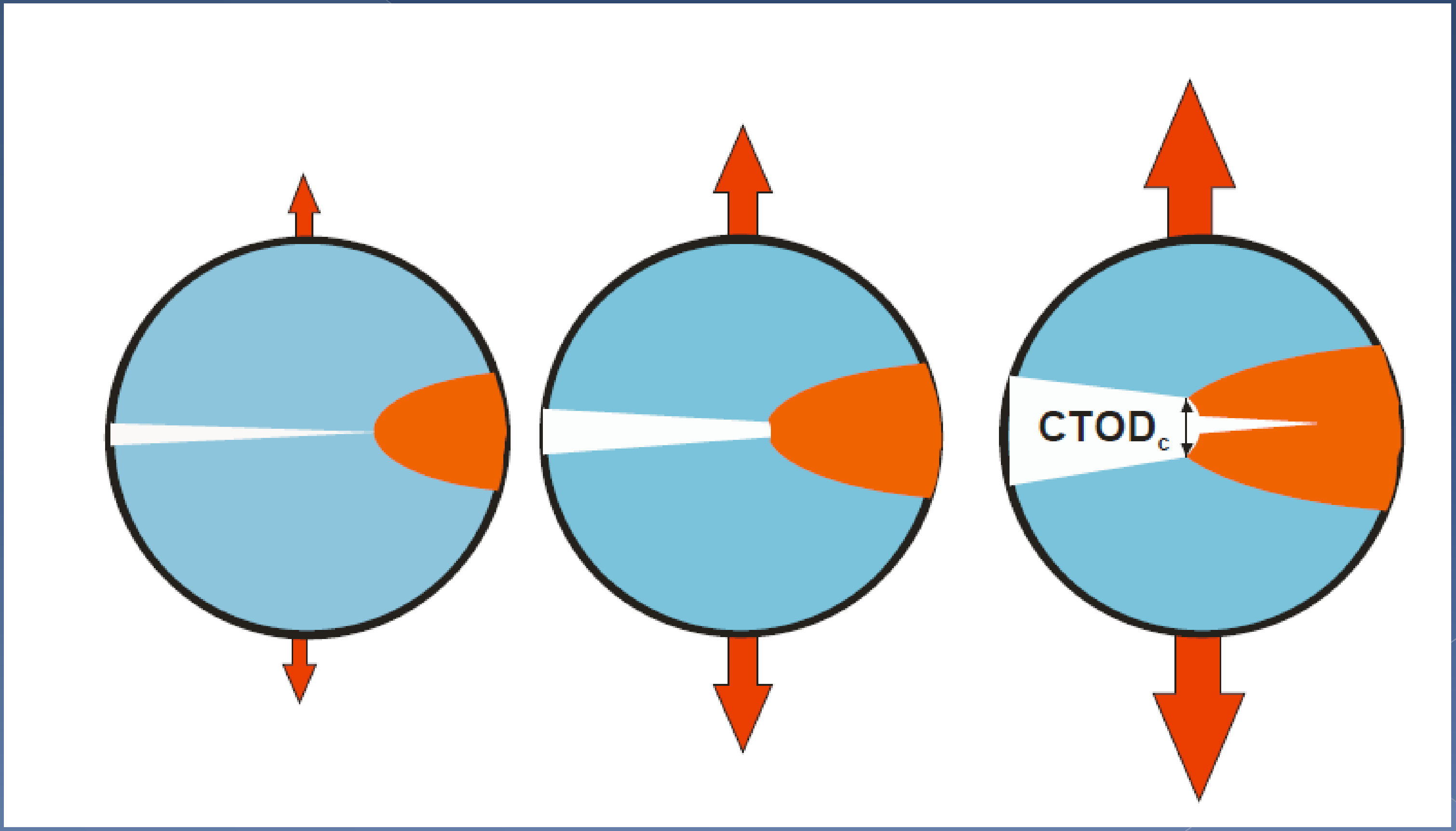
energy and the stress based criteria, for determining the onset of crack growth

$$J = \int_{\Gamma} \left(w dy - T_i \frac{\partial u_i}{\partial x} \right) ds$$



In cases where fracture is accompanied by substantial plastic deformation

an alternative description of the crack tip state has been established, designated the crack-tip-opening displacement (CTOD) approach”

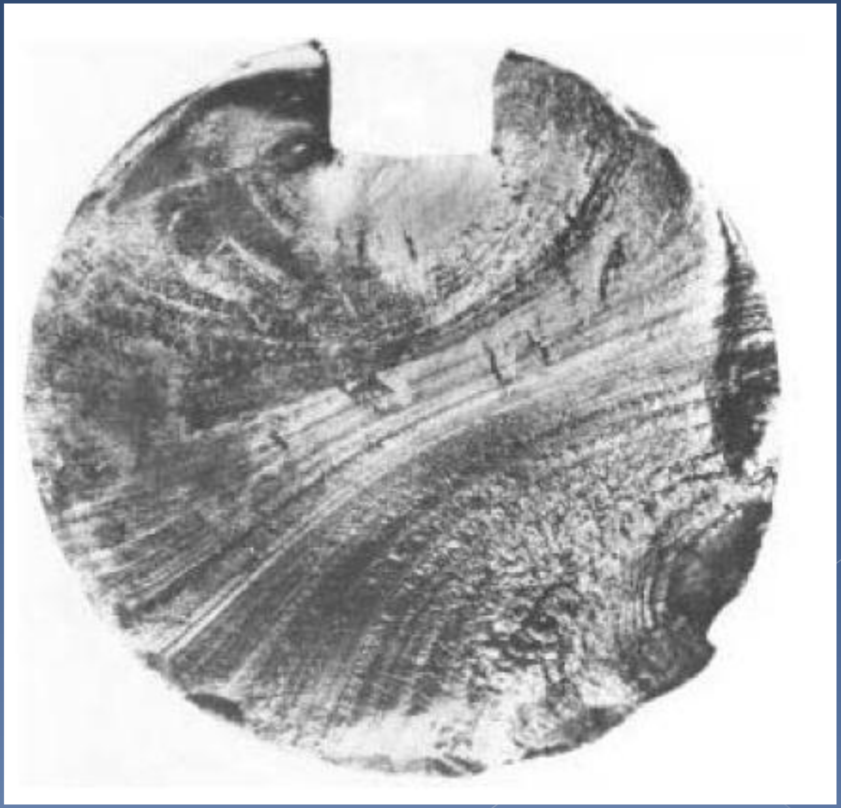


Fatigue

It has long been known that a component subjected to fluctuating stresses may fail at stress levels much lower than its monotonic fracture strength, due to a process called Fatigue.

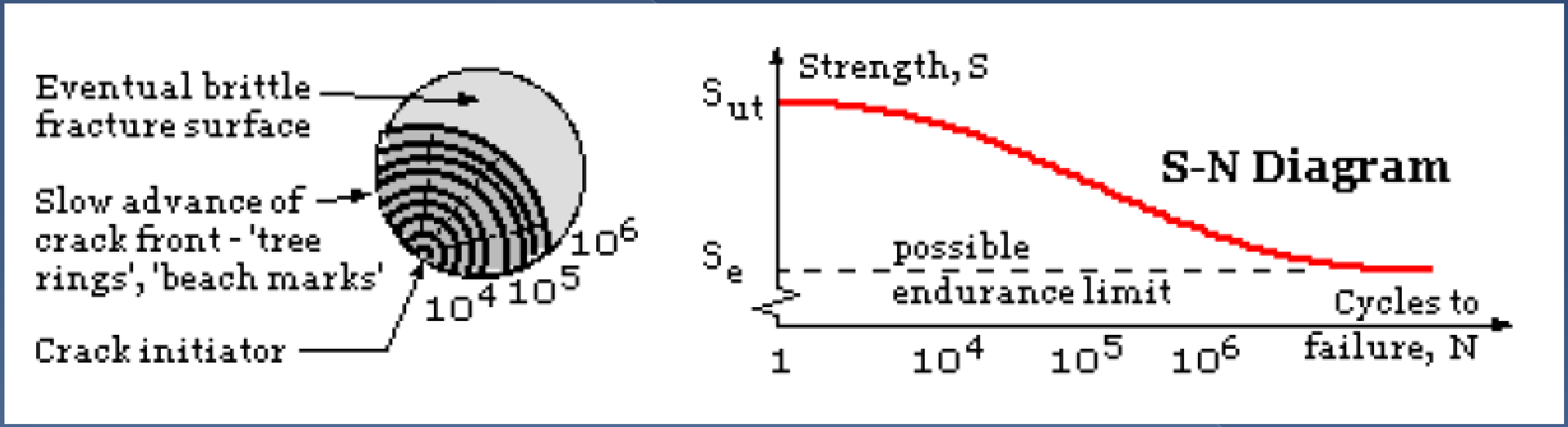
fatigue failure components

- **Crack Initiation**
- **Incremental Crack Growth**
- **Final Fracture**



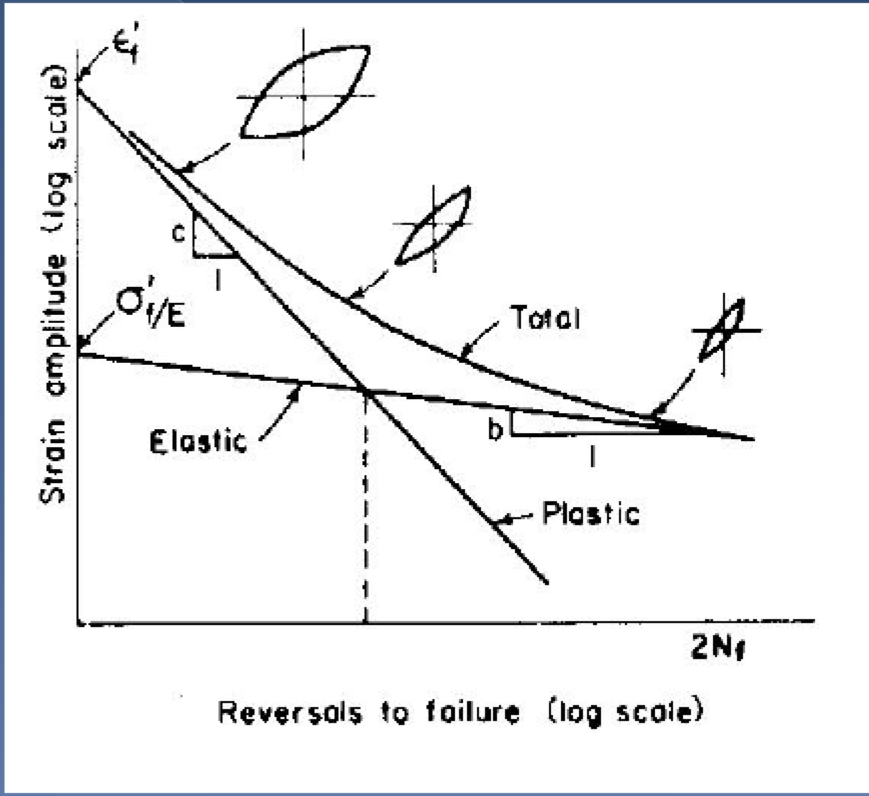
Classical Fatigue

The classical approach to fatigue, also referred to as Stress Controlled Fatigue or High Cycle Fatigue (HCF), through S/N or Wöhler diagrams, constitutes the basis of the SAFE LIFE philosophy in design against fatigue.



Low Cycle Fatigue

Based on the LCF local strain philosophy, fatigue cracks initiate as a result of repeated plastic strain cycling at the locations of maximum strain concentration.



Peridynamics approach

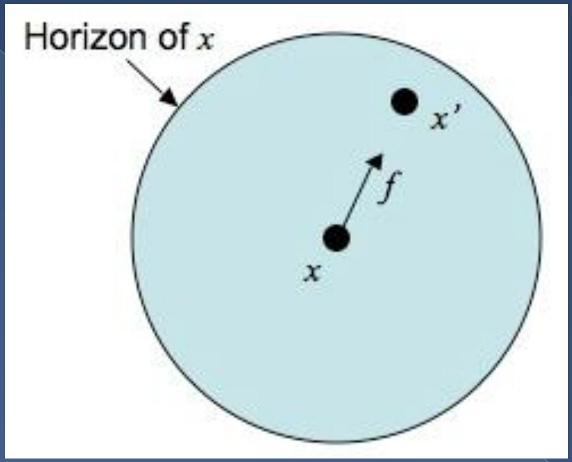
The main advantage of Peridynamics is that no a-priori knowledge about the crack initiation and propagation is required

Cracks are free to arise, grow and interact in every part of the structure, only according to physical and geometrical constraints.

- ❑ **Crack propagation in static problems**
- ❑ **Behavior of heterogeneous materials can be described**
- ❑ **Grid refinement techniques can be applied to increase the resolution of the analysis**

FUNDAMENTALS OF PERIDYNAMICS

- ✓ **Each point interacts with others within a finite distance δ called horizon**



- ✓ **The interaction between any pair of points exists even when they are not in contact.**

- ✓ **This physical interaction is referred to as “bond”**

The equations that govern the motion of the points of the structure assume the following form:

$$\rho \ddot{\mathbf{u}}_i(\mathbf{X}_j, t) = \int_{Hi} \mathbf{f}[\mathbf{u}(\mathbf{x}_j, t) - \mathbf{u}(\mathbf{x}_i, t), \mathbf{x}_j - \mathbf{x}_i] \mathrm{dV}_j + \mathbf{b}(\mathbf{X}_i, t) \qquad \forall \mathbf{x}_i \in B$$

- ❑ *H*i: Neighborhood of the point **x**i (a material node in the body *B*)
- ❑ δ: Horizon length
- ❑ **u:** Displacement vector field
- ❑ **b**: Body density force vector
- ❑ **f:** Pairwise force function

Under this assumption, and naming:

$$\boldsymbol{\xi} = \mathbf{x}_j - \mathbf{x}_i$$ → Initial relative position

$$\boldsymbol{\eta} = \mathbf{u}(\mathbf{x}_j, t) - \mathbf{u}(\mathbf{x}_i, t)$$ → Current relative displacement

The stretch s of the bond can be expressed as:

$$s = \frac{|\boldsymbol{\eta} + \boldsymbol{\xi}| - |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|}$$

- The pairwise (PW) force function depends on the bond stretch:
- When the stretch reaches a given limit value (Sc) the bond breaks

$$\mathbf{f} = c.s.\mu(\xi).\frac{\boldsymbol{\eta}+\xi}{|\boldsymbol{\eta}+\xi|}$$

C: Bond stiffness

$\mu(\xi)$: history-dependent scalar function which takes the values 1 or 0 depending on the bond status

$\mu(\xi) = 1$ → Active bond

$\mu(\xi) = 0$ → Broken bond

C varies with material elastic properties

$c = \dfrac{9.E}{\pi.t.\delta^3}$ → **2D plain stress**

$c = \dfrac{48.E}{5.\pi.t.\delta^3}$ → **2D plain strain**

$c = \dfrac{12.E}{\pi.\delta^4}$ → **3D**

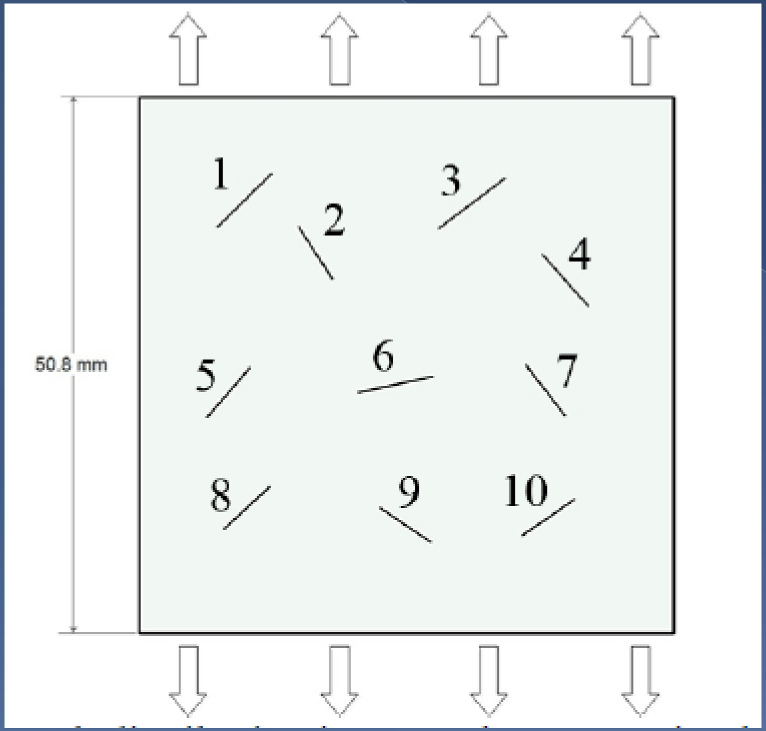
The limit stretch s_c is estimated by the evaluation of the energy required to break all the bonds that initially connect points in the opposite sides of a fracture surface.

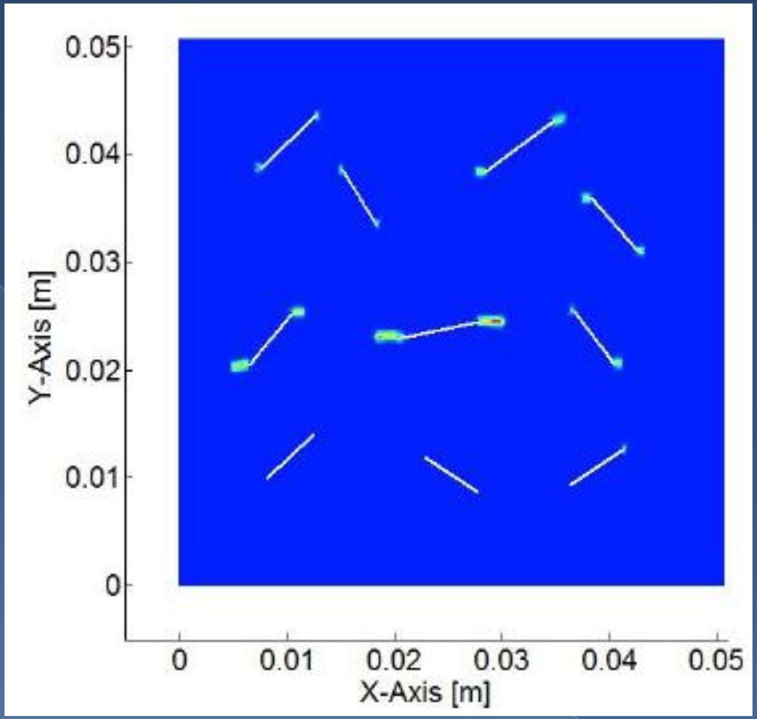
$$
\left\{
\begin{array}{l}
s_c = \sqrt{\dfrac{4\pi G_{Ic}}{9E\delta}} \Rightarrow \textbf{2D plain stress} \\[2ex]
s_c = \sqrt{\dfrac{5\pi G_{Ic}}{12E\delta}} \Rightarrow \textbf{2D plain strain} \\[2ex]
s_c = \sqrt{\dfrac{5G_{Ic}}{6E\delta}} \Rightarrow \textbf{3D}
\end{array}
\right.
$$

G_{Ic} : Critical energy release rate

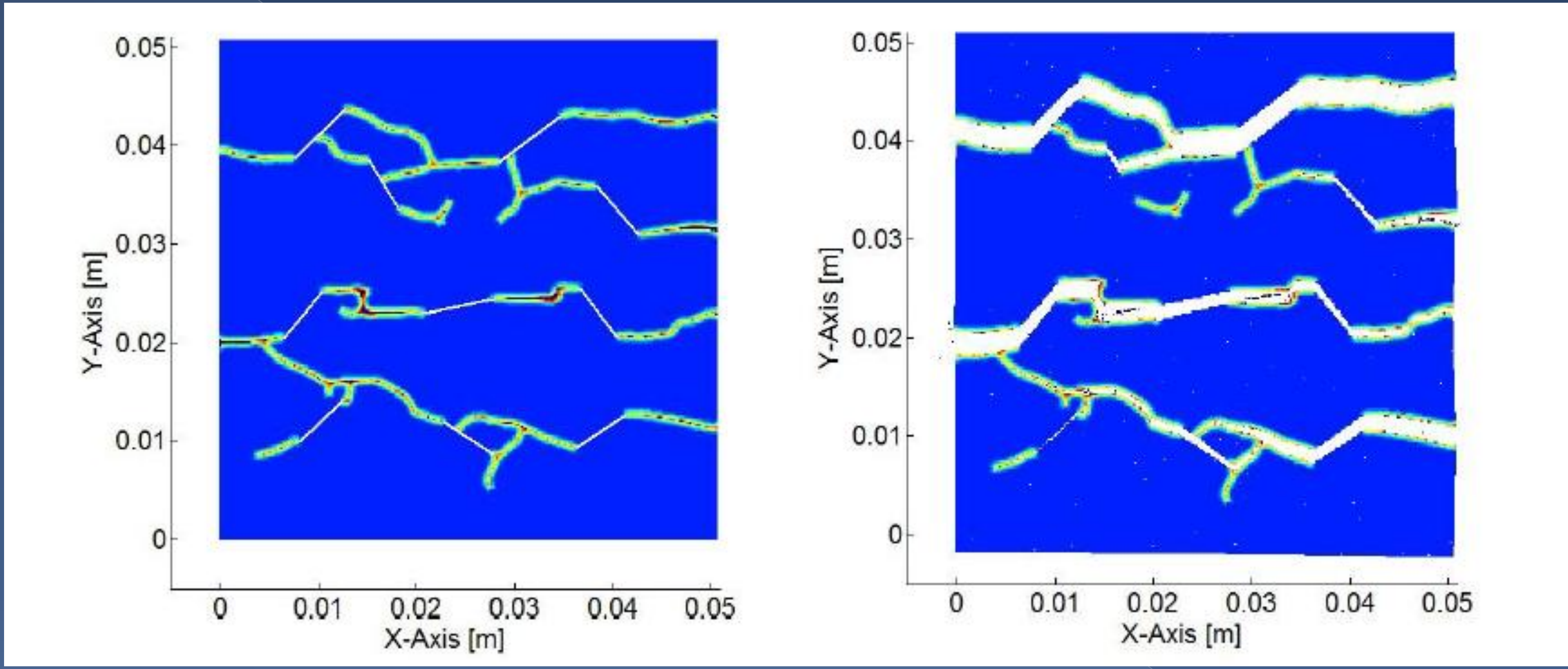
Examples which solved by previous researchers

DYNAMIC PROPAGATION OF RANDOM DISTRIBUTED CRACKS



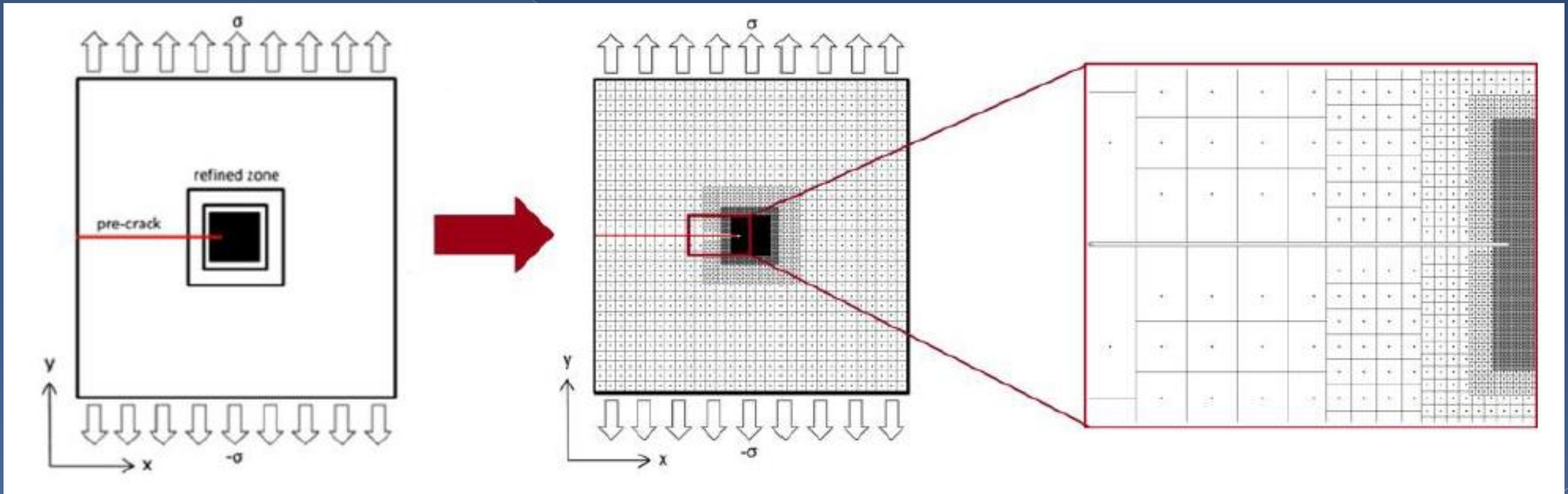


DYNAMIC PROPAGATION OF RANDOM DISTRIBUTED CRACKS

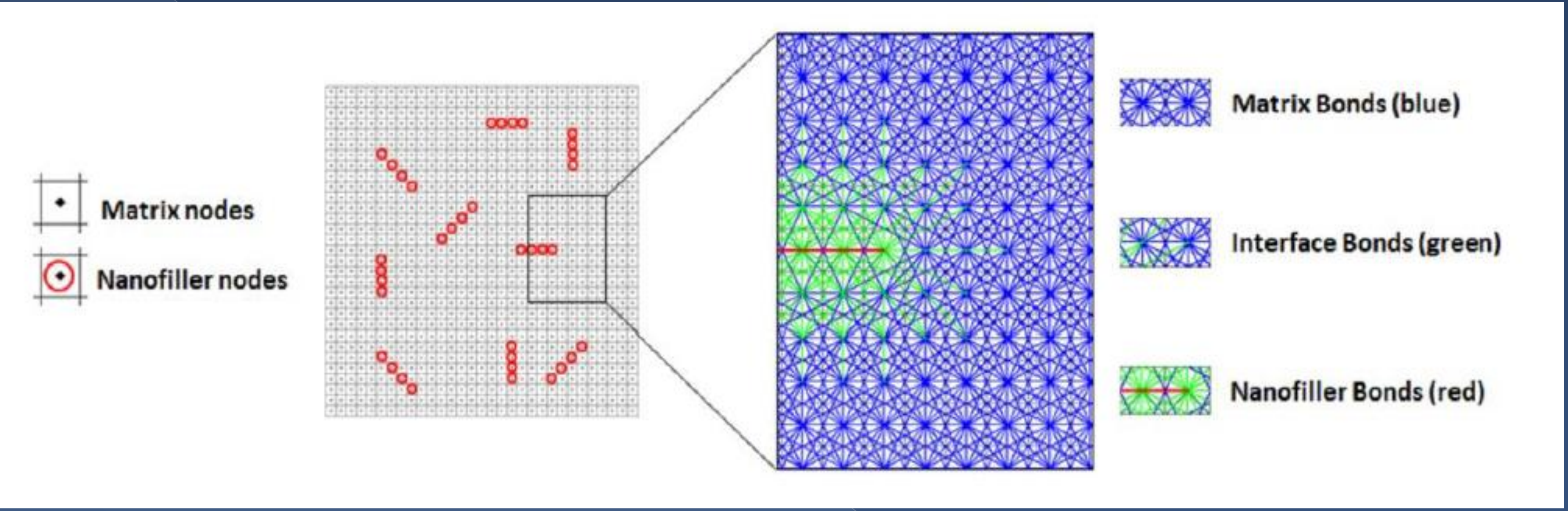


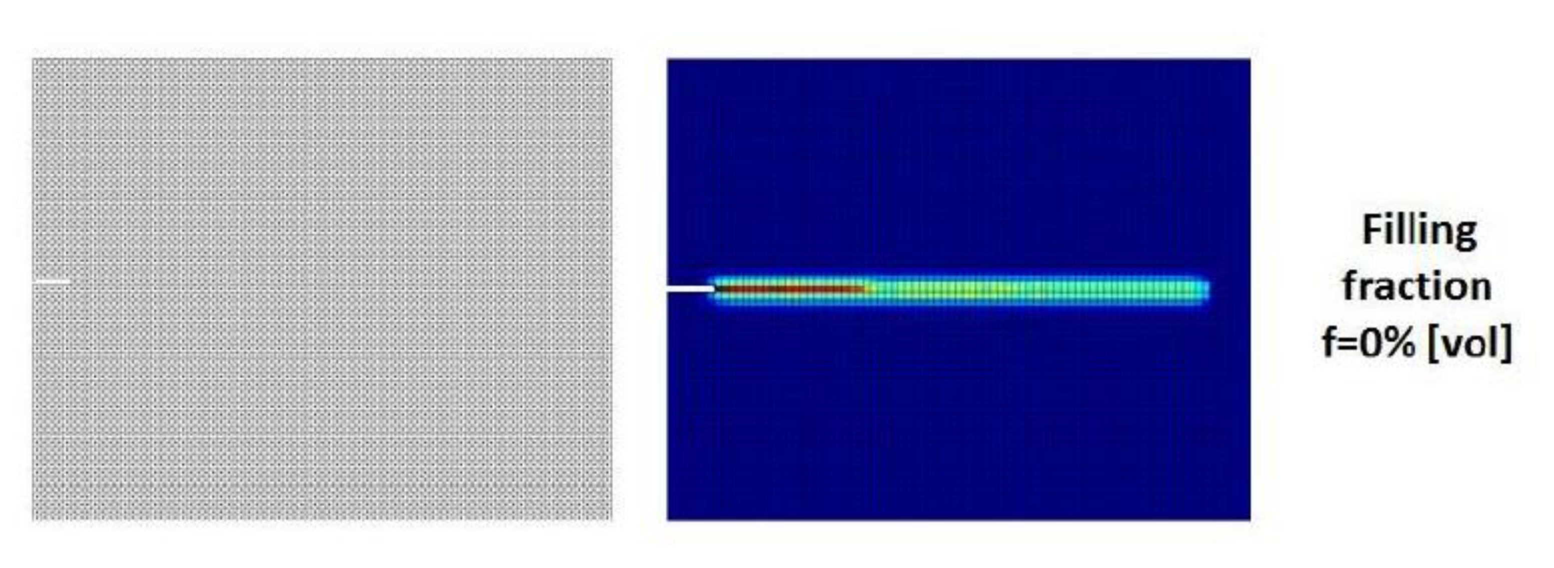
MULTISCALE MODELING

Model of a pre-cracked plate subject to a traction load, on the right a detail of the refined region where the crack propagation takes place.

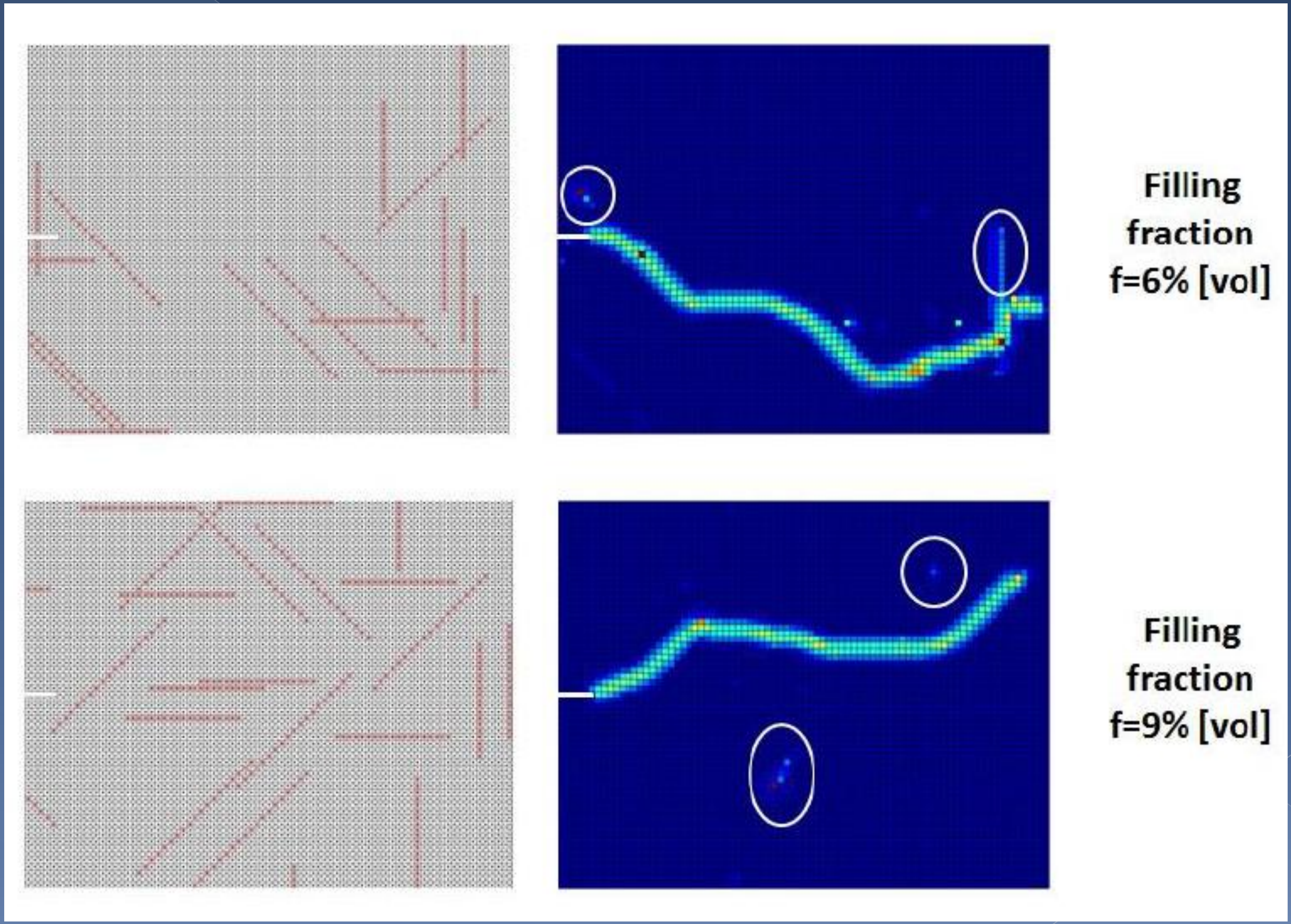


A detailed view of the different properties assigned to the bonds belonging to the nanoscale region





Crack propagation paths obtained for different filling fractions. The damaged zones marked with a circle and far from the crack are due to the agglomeration weakening effect. The nano-fibers disposition is shown on the left.



Conclusion

- The theory is able to solve problems in presence of discontinuities (as cracks, voids, inclusions)
- Impact & fragmentation (dynamics crack)
- Static and fatigue phenomena
- Problems with Large Elastic Deformation
- Coupled problems (cracks propagation due to stress-corrosion and environmental effect)

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