Università degli Studi di Padova

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A HYBRID NONLOCAL-LOCAL MESHLESS APPROACH FOR DYNAMIC CRACK PROPAGATION

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Introduction:

- Peridynamic [1] is a new developed nonlocal continuum theory well suited to model crack propagation .
- The main advantage of Peridynamics is that no a-priori knowledge about the crack initiation is required.
- Crack is free to arise and grow in every part of the structure, only following physical and geometrical constraints.

Stewart Silling

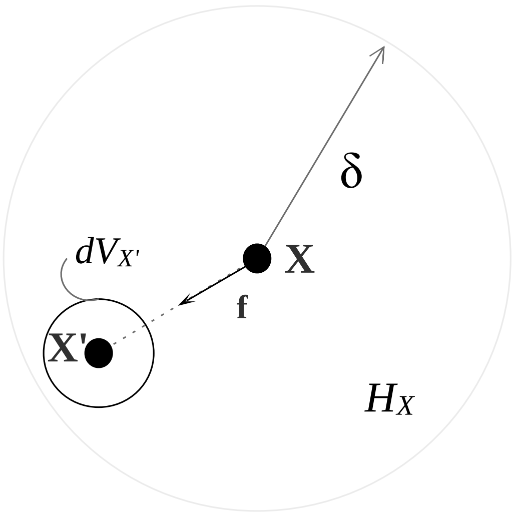
[1]: Silling, S.A.: Reformulation of elasticity theory for discontinuities and long-range forces. J. Mech. Phys. Solids. 48, 175–209 (2000).

Introduction:

The Peridynamic formulation:

The equation of motion:

$$\int_{H_X} \mathbf{f}(\mathbf{u}(\mathbf{X}',t)-\mathbf{u}(\mathbf{X},t),\mathbf{X}-\mathbf{X}')dV_{X'} + \mathbf{b}(\mathbf{X},t) = \rho\ddot{\mathbf{u}}(\mathbf{X},t)$$



The pairwise force function should satisfy:

$$\mathbf{f}(-\boldsymbol{\eta},-\boldsymbol{\xi}) = \mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) \longrightarrow$$ conservation of linear momentum

$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) \times (\boldsymbol{\eta}+\boldsymbol{\xi}) = \mathbf{0} \longrightarrow$$ conservation of angular momentum

$$H_X = \{\mathbf{X}' \in \Omega, |\mathbf{X}'-\mathbf{X}| < \delta\}$$

$$\boldsymbol{\xi} = \mathbf{X}' - \mathbf{X}$$

$$\boldsymbol{\eta} = \mathbf{u}(\mathbf{X}',t) - \mathbf{u}(\mathbf{X},t)$$

The main idea of the present study:

Rationale:

- Meshless Peridynamic models due to nonlocal integration involved in the theory and the interaction of each integration point with multiple neighbors, are computationally more demanding than the majority of other meshless methods based on classical elasticity.

Our idea:

- The main idea of the present study is taking full advantage of both theories by coupling a meshless Peridynamic method [2] with simple and computationally cheap meshless finite point method (FPM) [3].

[2]: Silling, S.A., Askari, E.: A meshfree method based on the peridynamic model of solid mechanics. Comput. Struct. 83, 1526–1535 (2005).

[3]: Oñate, E., Perazzo, F., Miquel, J.: A finite point method for elasticity problems. Comput. Struct. 79, 2151–2163 (2001).

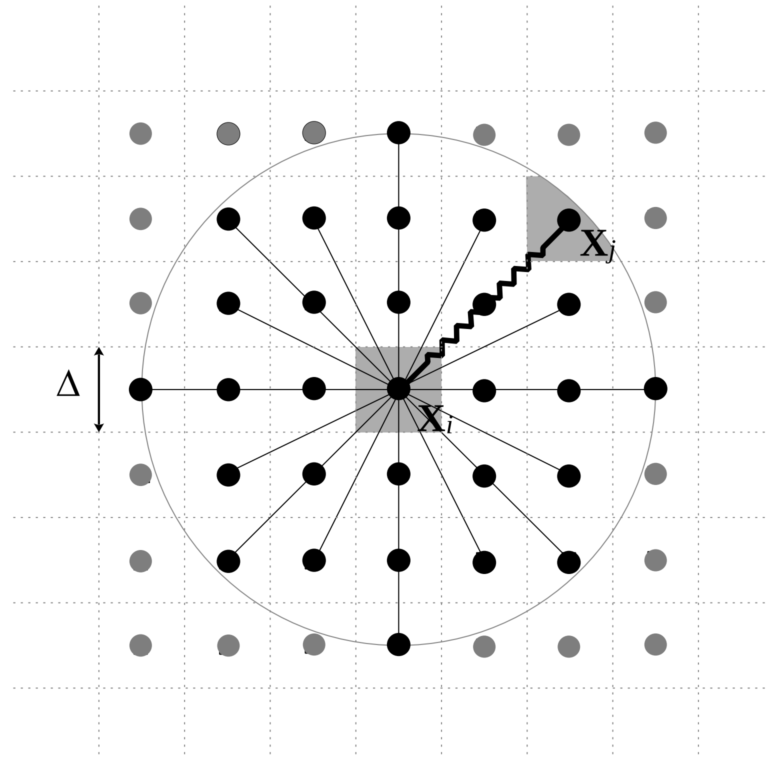
Discretized numerical methods:

Peridynamic:

$$\rho \ddot{\mathbf{u}}_i^n = \sum_j \mathbf{f}(\mathbf{u}_i^n - \mathbf{u}_j^n, \mathbf{X}_i - \mathbf{X}_j)\underbrace{\beta_j}_{\text{Volume correction factor}} V_j + \mathbf{b}_i^n$$

$$\underbrace{\mathbf{k}_{ij}}_{\text{Truss element}} = \frac{c}{|\boldsymbol{\xi}|}\beta_j V_i V_j \begin{bmatrix} \xi_1^{\,2} & \xi_1\xi_2 & -\xi_1^{\,2} & -\xi_1\xi_2 \\ & \xi_2^{\,2} & -\xi_1\xi_2 & -\xi_2^{\,2} \\ & & \xi_1^{\,2} & \xi_1\xi_2 \\ & & & \xi_2^{\,2} \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{ij}^{11} & \mathbf{k}_{ij}^{12} \\ \mathbf{k}_{ij}^{21} & \mathbf{k}_{ij}^{22} \end{bmatrix}$$

$$\boldsymbol{\xi} = (\xi_1, \xi_2)$$



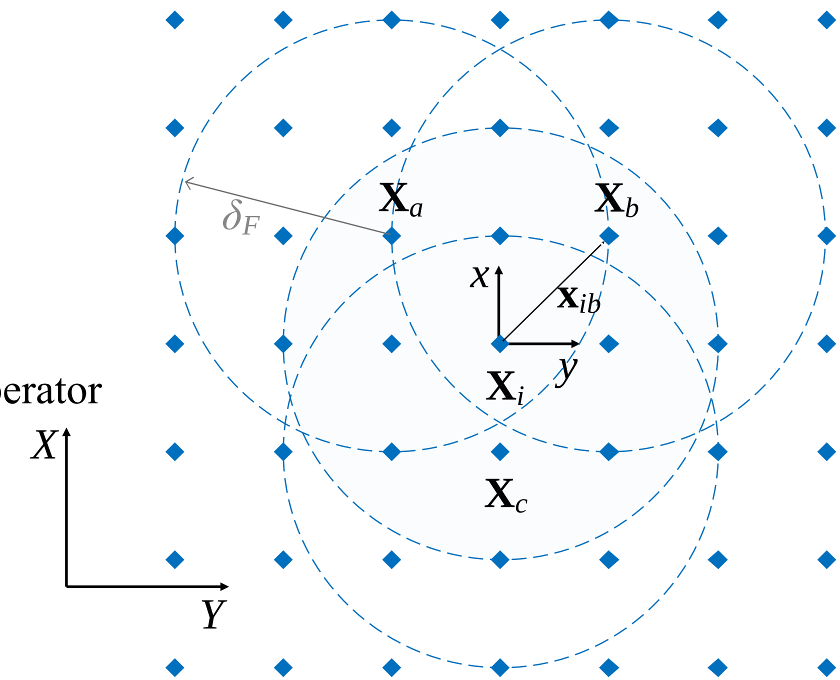
A generic horizon in discrete form

Discretized numerical methods:

FPM:

Governing equation:

$$\mathbf{S}^{\mathrm{T}}\mathbf{D}\mathbf{S}\mathbf{u}(\mathbf{X},t)+\mathbf{b}(\mathbf{X},t)=\rho\ddot{\mathbf{u}}(\mathbf{X},t)$$



$$\mathbf{S}=\begin{bmatrix}\partial/\partial X & 0 & \partial/\partial Y \\ 0 & \partial/\partial Y & \partial/\partial X\end{bmatrix}^T \longrightarrow$$ Differential operator

$$\mathbf{D}=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & (1-2\nu)/2\end{bmatrix} \longrightarrow$$ Plane stress condition

Discretized numerical methods:

FPM:

Approximation Scheme:

$$\hat{u}^n(\mathbf{x}) = \sum_{l=1}^{m_p} p_l(\mathbf{x})\alpha_l^n = \mathbf{p}^T(\mathbf{x})\boldsymbol{\alpha}^n, \quad \mathbf{x} = (x, y) \qquad \mathbf{p}(\mathbf{x}) = \left(1, x, y, x^2, y^2, xy\right)^T, \quad \text{for } m_p = 6$$

$$\mathbf{x}^R = \begin{pmatrix} \vdots \\ \mathbf{x}_{ia} \\ \mathbf{x}_{ib} \\ \mathbf{x}_{ic} \\ \vdots \end{pmatrix}, \quad (\mathbf{x}_{ij} = \mathbf{X}_j - \mathbf{X}_i) \;\&\; \mathbf{X}_j \in C_{X_i}$$

$$\hat{\mathbf{u}}_R^n = \begin{pmatrix} \vdots \\ \hat{u}_a^n \\ \hat{u}_b^n \\ \hat{u}_c^n \\ \vdots \end{pmatrix} = \begin{bmatrix} \vdots \\ \mathbf{p}^T(\mathbf{x}_{ia}) \\ \mathbf{p}^T(\mathbf{x}_{ib}) \\ \mathbf{p}^T(\mathbf{x}_{ic}) \\ \vdots \end{bmatrix} \boldsymbol{\alpha}^n = \mathbf{C}\boldsymbol{\alpha}^n$$

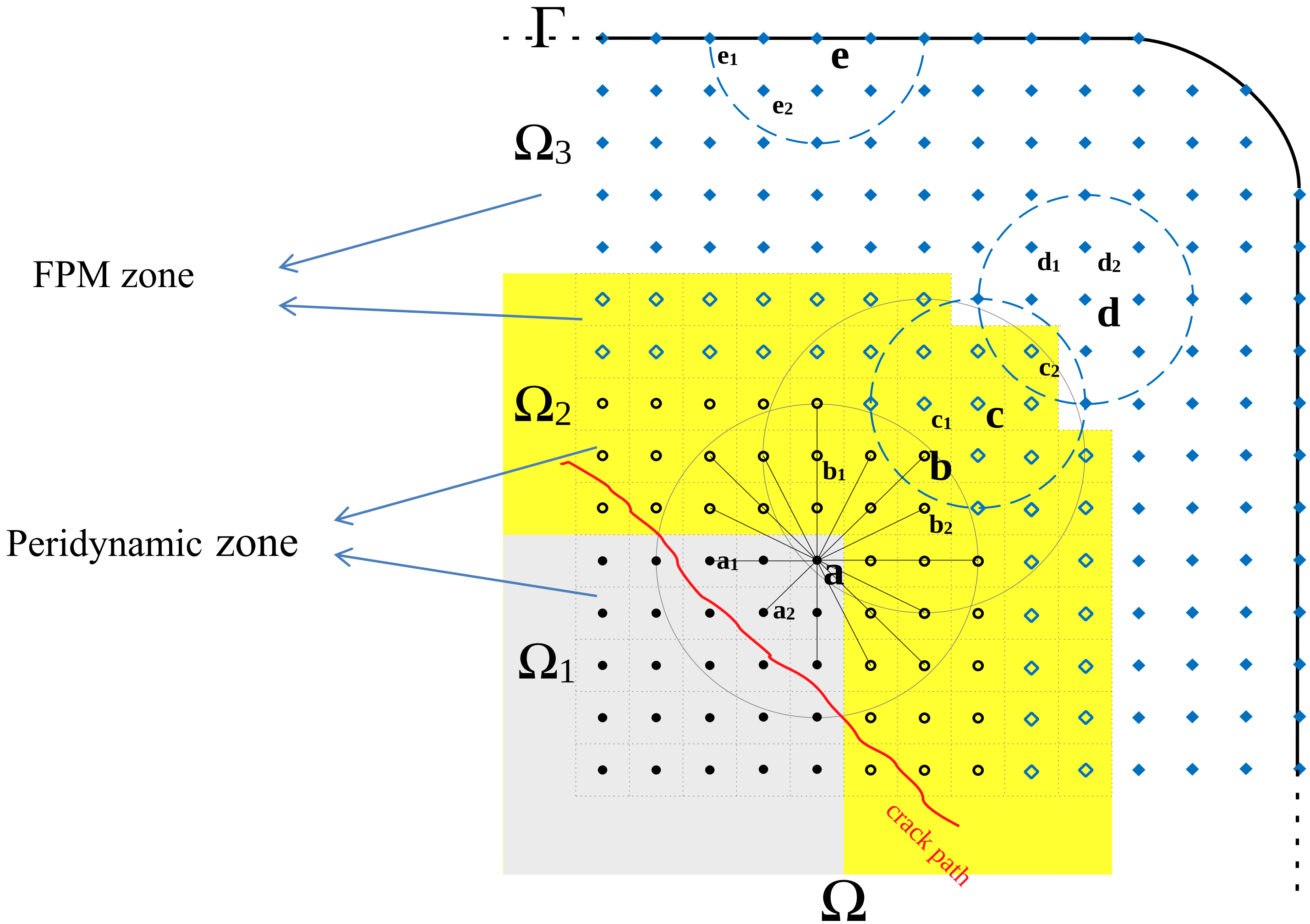
Cloud moment matrix

Least square

$$\hat{u}^n(\mathbf{x}) = \mathbf{p}^T \mathbf{C}^{-1} \hat{\mathbf{u}}_R^n = \sum_{j=1}^{n_R} N_j \hat{u}_j^n$$

Shape functions

Coupling technique:



Coupling technique:

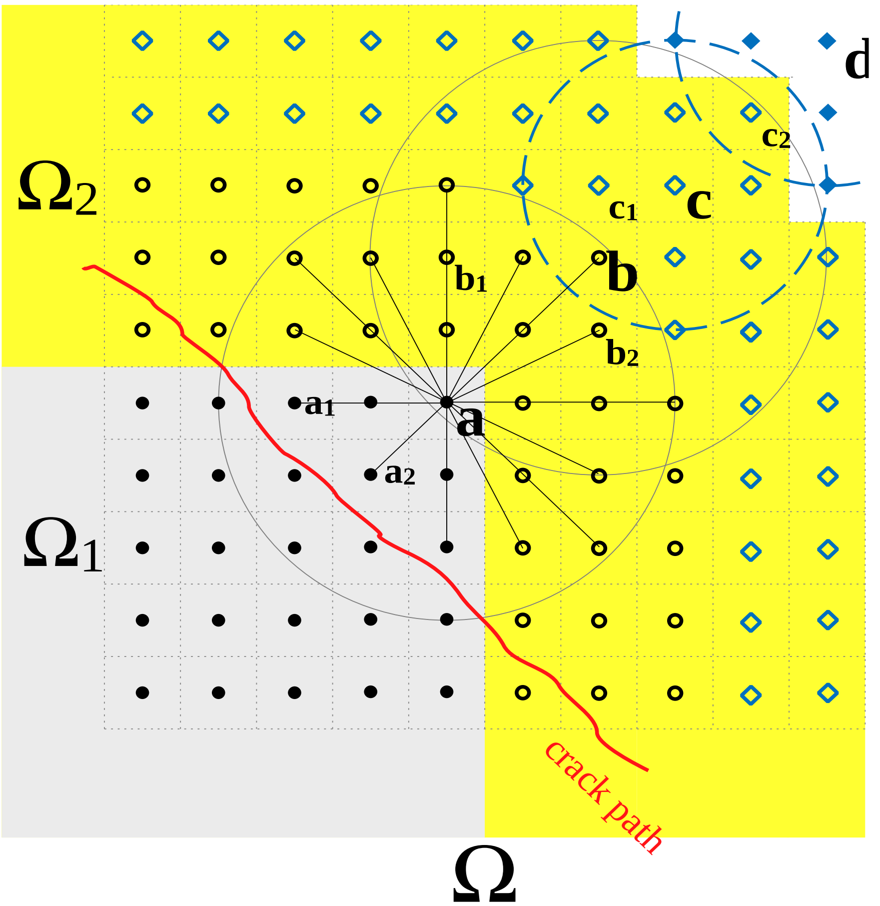
Peridynamic zone:

Layers “a” and “b”:

$$\rho \ddot{\mathbf{u}}_a^n = \ldots \mu_{aa_1}^n \hat{\mathbf{f}}_{aa_1}^n + \mu_{aa_2}^n \hat{\mathbf{f}}_{aa_2}^n + \ldots$$
$$\mu_{ab}^n \hat{\mathbf{f}}_{ab}^n + \mu_{ab_1}^n \hat{\mathbf{f}}_{ab_1}^n + \mu_{ab_2}^n \hat{\mathbf{f}}_{ab_2}^n + \ldots + \mathbf{b}_a^n$$

$$\rho \ddot{\mathbf{u}}_b^n = \ldots \mu_{ba}^n \hat{\mathbf{f}}_{ba}^n + \ldots + \mu_{bb_1}^n \hat{\mathbf{f}}_{bb_1}^n + \mu_{bb_2}^n \hat{\mathbf{f}}_{bb_2}^n + \ldots$$
$$+ \mu_{bc}^n \hat{\mathbf{f}}_{bc}^n + \mu_{bc_1}^n \hat{\mathbf{f}}_{bc_1}^n + \mu_{bc_2}^n \hat{\mathbf{f}}_{bc_2}^n + \ldots \mathbf{b}_b^n$$

$$\hat{\mathbf{f}}_{ij}^n = \mathbf{k}_{ij}^{11} \mathbf{u}_i^n + \mathbf{k}_{ij}^{21} \mathbf{u}_j^n$$



Coupling technique:

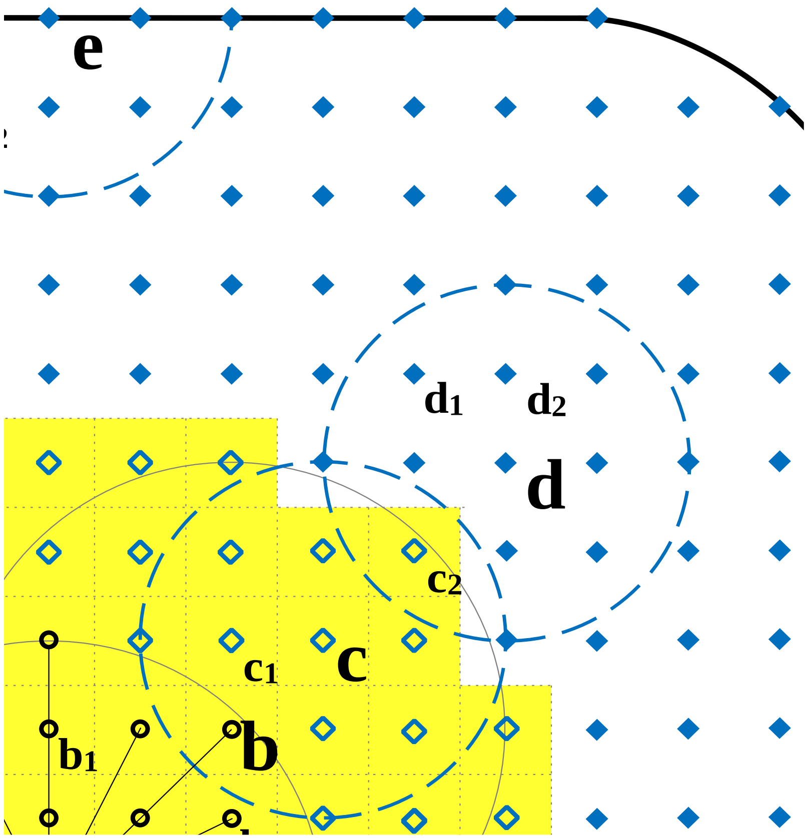
Finite Point zone:

Layers “c” and “d”:

$$\rho \ddot{\mathbf{u}}_c^n = \ldots + \overline{\mathbf{f}}_{cb} + \ldots + \overline{\mathbf{f}}_{cc} + \overline{\mathbf{f}}_{cc_1} + \overline{\mathbf{f}}_{cc_2} + \ldots \mathbf{b}_c^n$$

$$\rho \ddot{\mathbf{u}}_d^n = \ldots + \overline{\mathbf{f}}_{dd} + \overline{\mathbf{f}}_{dd_1} + \overline{\mathbf{f}}_{dd_2} \ldots \mathbf{b}_d^n$$

$$\overline{\mathbf{f}}_{ij} = (\mathbf{S}^{\mathrm{T}}\mathbf{DSN}_j)\Big|_{X_i} \mathbf{u}_j^n$$



Coupling technique:

Ghost force Test:

a Dirichlet boundary condition is imposed on all the boundaries as:

$$\mathbf{u} = \langle 1,1 \rangle^T$$

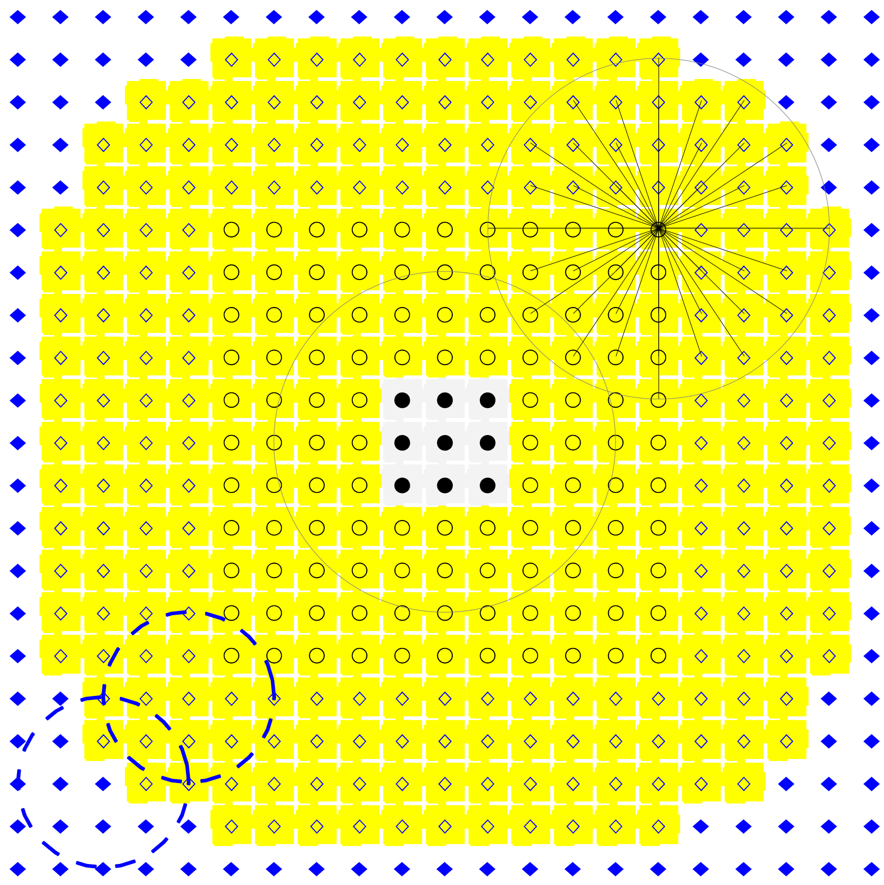
Norm of error for different layers:

$$e_a = 1.44 \times 10^{-15}$$

$$e_b = 1.17 \times 10^{-15}$$

$$e_c = 1.30 \times 10^{-15}$$

$$e_d = 8.67 \times 10^{-15}$$

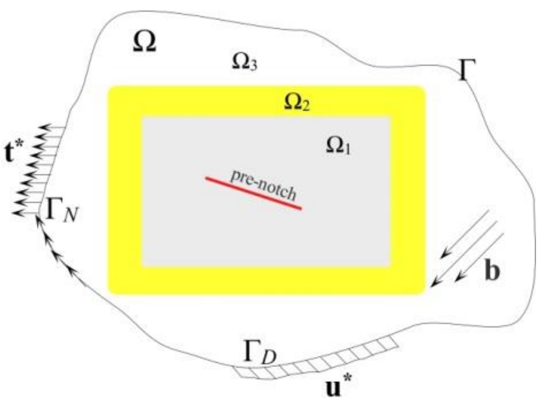


Coupling technique:

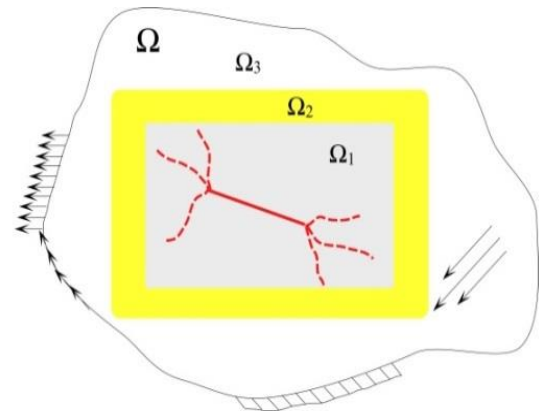
Partitioning configurations:

Static configurations:

$t = t_0$

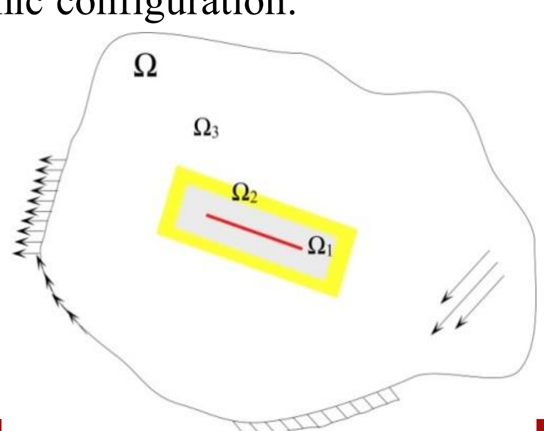


$t = t_f$

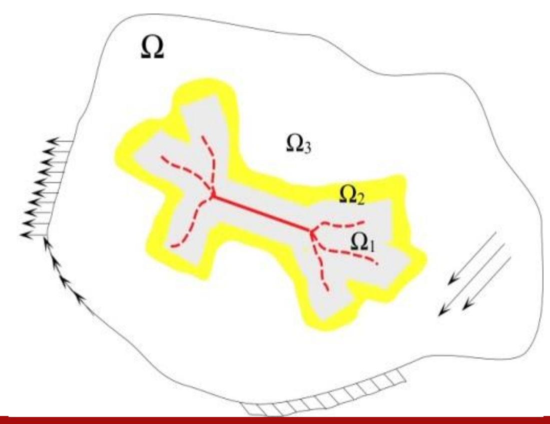


Adaptive and Dynamic configuration:

$t = t_0$

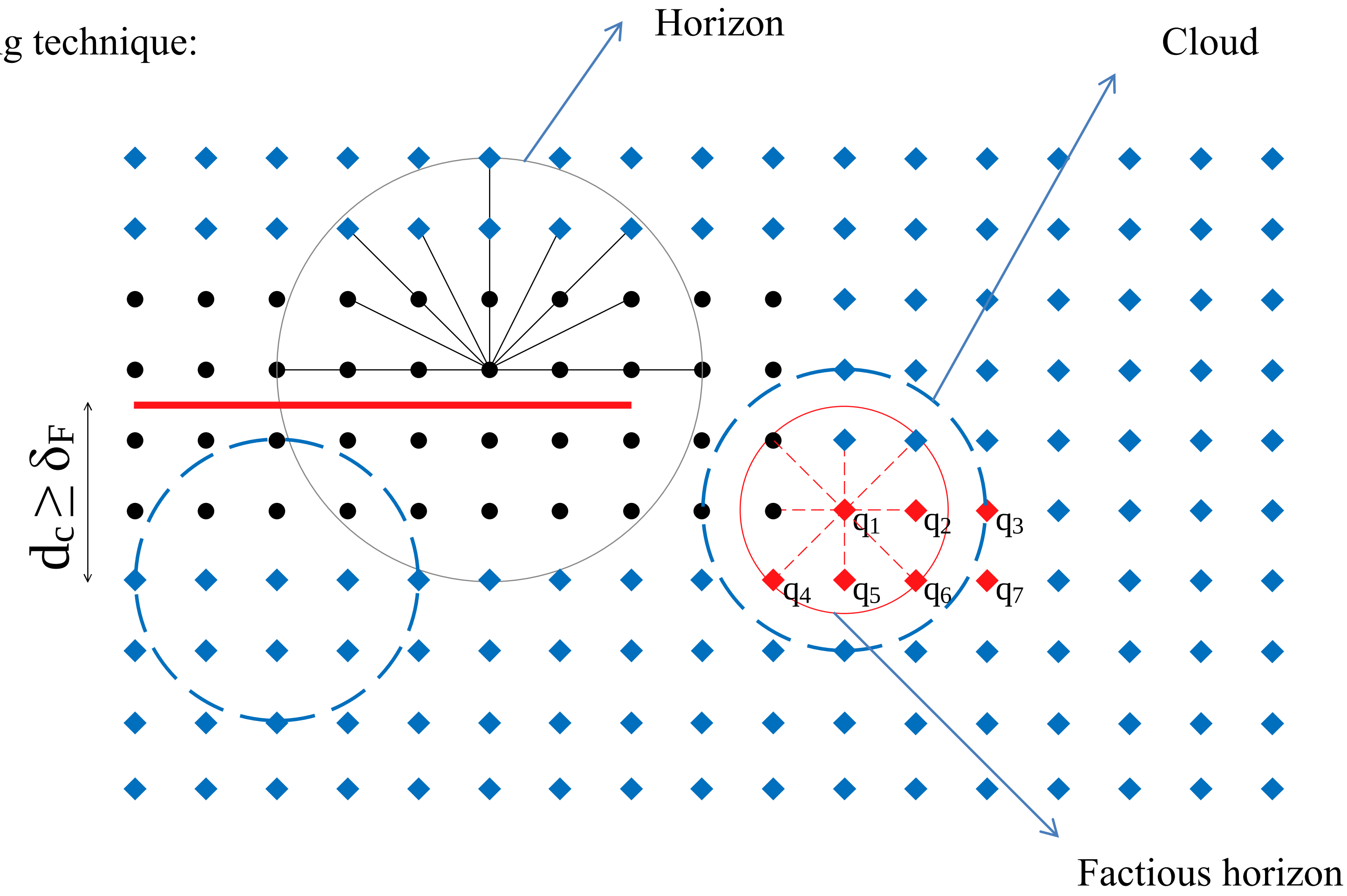


$t = t_f$



Coupling technique:

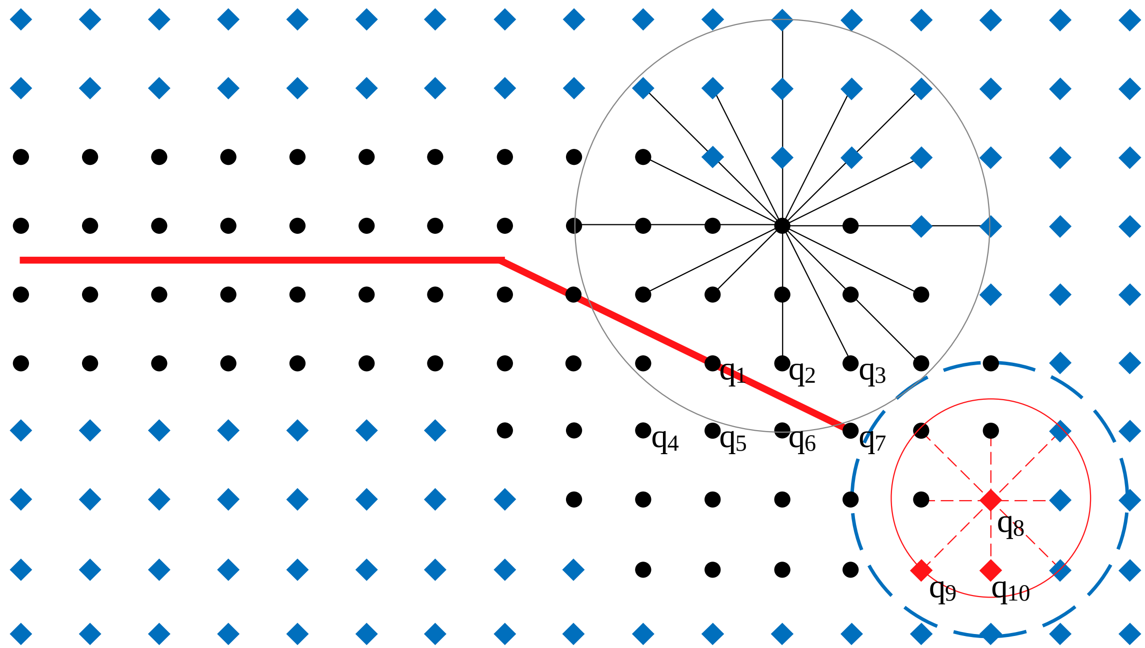
The switching technique:



Coupling technique:

The switching technique:

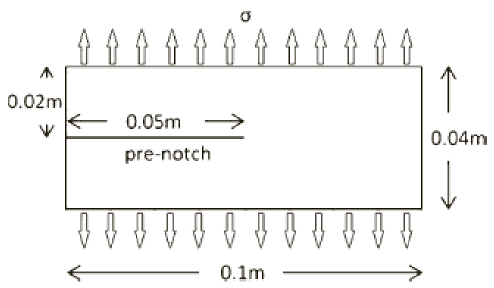
The switching criterion:



$$\chi s_0 \leq s'^{n}_{ij} < s_0, \qquad 0 \leq \chi \leq 1 \qquad s'^{n}_{ij} = \frac{\left|\mathbf{u}_i^n - \mathbf{u}_j^n\right|}{\left|\mathbf{x}_{ij}\right|}$$

Crack growth in a pre-cracked plate:

The problem parameters:

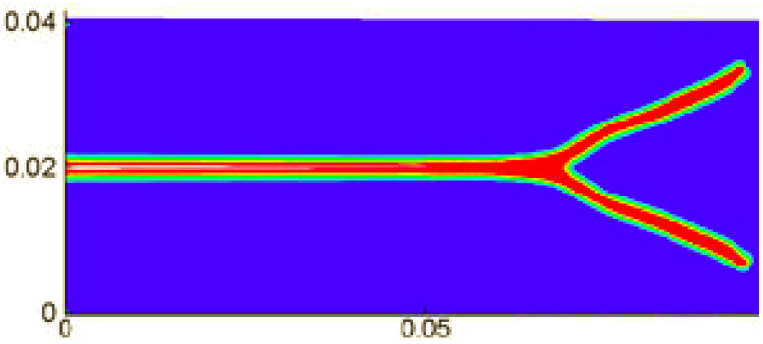


Material: Duran 50 glass

Duration: 46 μs

σ = 12 MPa

The solution based on a pure Peridynamic model with 64,000 nodes:



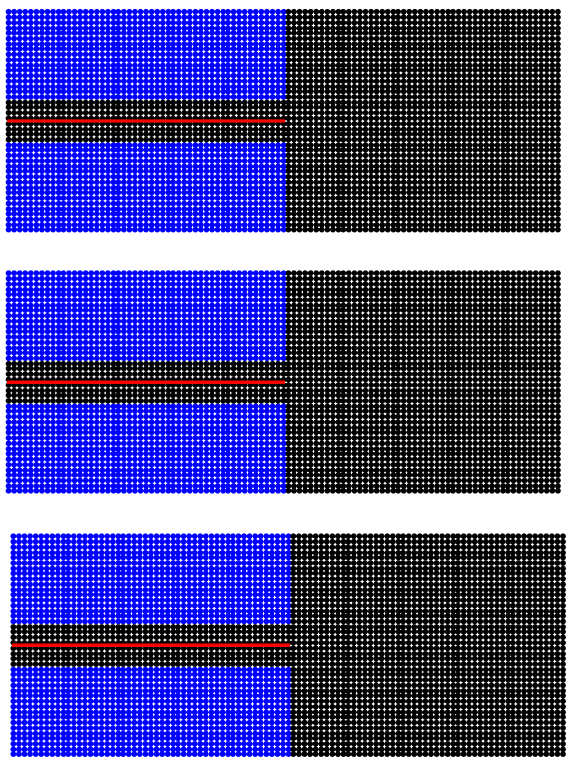
Ha & Bobaru 2010

[4]: Ha, Y.D., Bobaru, F.: Studies of dynamic crack propagation and crack branching with peridynamics. Int. J. Fract. 162, 229–244 (2010).

Crack growth in a pre-cracked plate:

Solution:

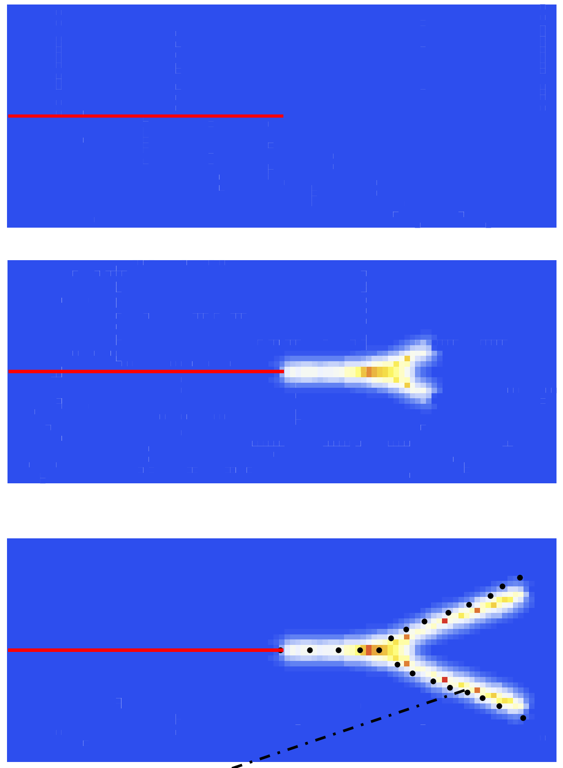
Static configuration Model I: (4242 nodes)



$t = 0$ s

$t = 32.2$ μs

$t = 46.2$ μs

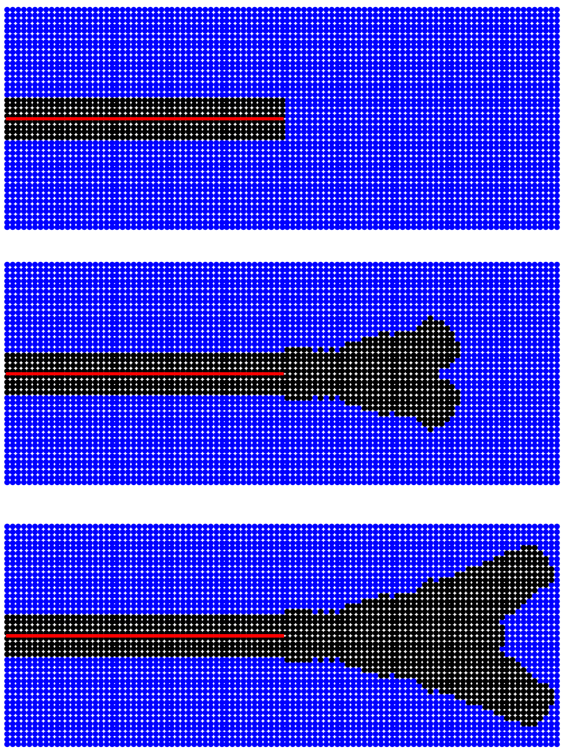


Ha & Bobaru 2010

Crack growth in a pre-cracked plate:

Solution:

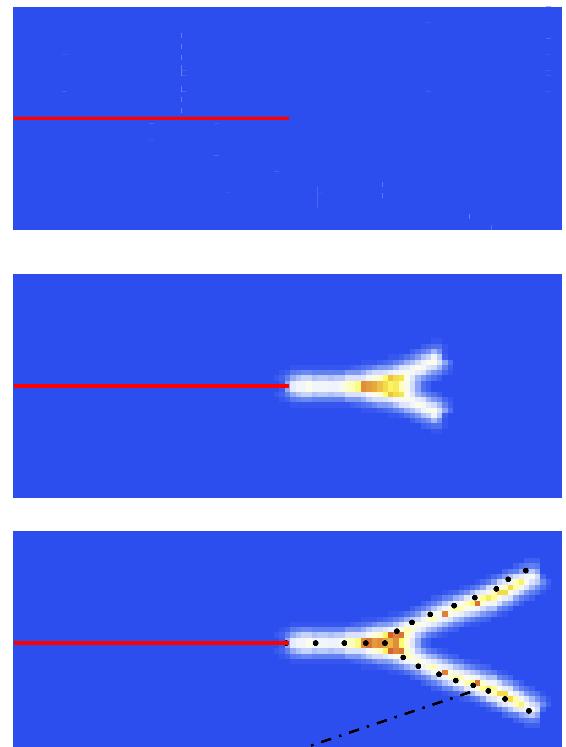
Dynamic configuration Model II: (4242 nodes) χ=0.6



t = 0 s

t = 32.2 μs

t = 46.2 μs

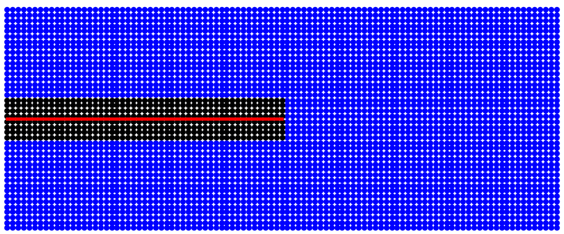


Ha & Bobaru 2010

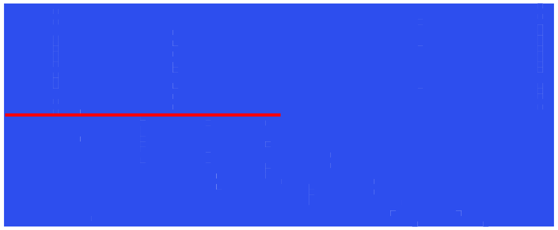
Crack growth in a pre-cracked plate:

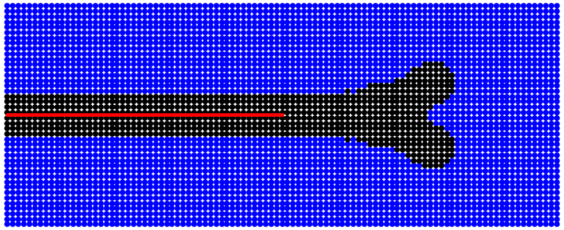
Solution:

Dynamic configuration Model II: (4242 nodes) $\chi=0.8$

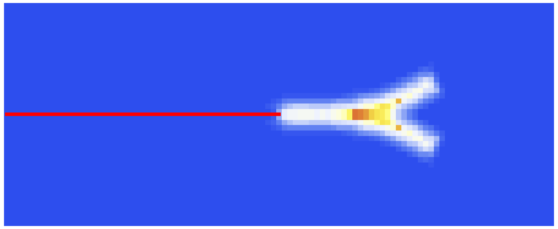


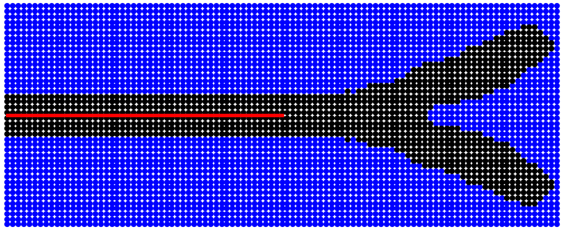
$t = 0$ s



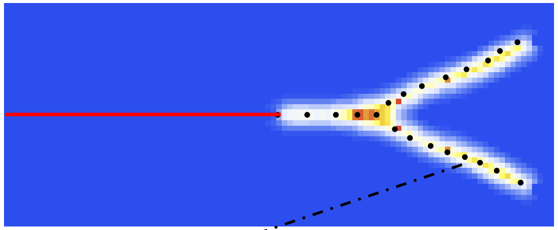


$t = 32.2\ \mu s$





$t = 46.2\ \mu s$



Ha & Bobaru 2010

Crack growth in a pre-cracked plate:

Discussion:

Model	Portion of Peridynamic nodes at at t=0	Portion of Peridynamic nodes at t=46 µs	CPU Time (s)
Model I	59 %	59 %	2362.78
Model II	9 %	29.4 %	1256.80
Model III	9 %	25.69 %	1207.52

System Properties:

- Intel® Core™ i7-3770 CPU @ 3.40 GHz
- Ram: 6 GB
- Windows 7 Professional

Remarks:

- A new coupling technique to couple Peridynamic with FPM is introduced.
- The coupling is done in a complete meshless style preserving the originality of the both formulations.
- The coupling is done through which no ghost forces emerge in the solution domain.
- The method is suitable to be applied for dynamic crack propagation.

Publications in Journals:

[1]: Galvanetto, U., Mudric, T., Shojaei, A., & Zaccariotto, M. (2016). An effective way to couple FEM meshes and Peridynamics grids for the solution of static equilibrium problems. *Mechanics Research Communications*, *76*, 41-47.

[2]: Shojaei, A., Zaccariotto, M., & Galvanetto, U. (2017). Coupling of 2D discretized Peridynamics with a meshless method based on classical elasticity using switching of nodal behavior. (Revision Submitted)

[3]: Shojaei, A., Mudric, T., Zaccariotto, M., & Galvanetto, U. (2017). A coupled meshless finite point/Peridynamic method for 2D dynamic fracture analysis. (Under Review)

Our plan for the third year:

Extending the work for practical 3D challenging problems.

Thank you very much