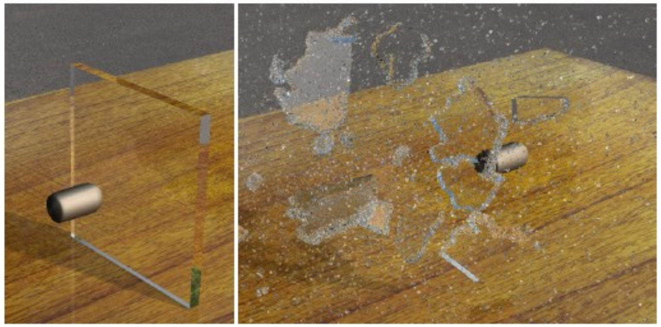
Dipartimento
di Ingegneria Industriale
Via Venezia 1, 35131 Padova

PhD Course in Space Sciences, Technologies and Measurements

Sciences and Technologies for Aeronautics and Satellite Applications (STASA)

XXIX CICLO

Adaptive grid refinement and scaling techniques applied to peridynamics



By J.A. Levine et al.

Supervisor: prof. Ugo Galvanetto

Co-supervisor: ing. Mirco Zaccariotto

Doctoral student: Daniele Dipasquale

Email addresses: daniele.dipasquale@studenti.unipd.it/ da_iele@yahoo.it

Contents

- **Overview of the bond-based peridynamic theory and its numerical implementation**
  - Fundamentals
  - Scaling and dual-horizon concept
  - Numerical discretization with the meshfree method
- **Adaptive Grid Refinement and Scaling algorithm (AGRS)**
  - Triggers based on the energy and damage
  - Working principle of AGRS
- **Activities carried out during the doctoral period**
  - Static tests of refinement/scaling
  - Addressing grid sensitivity in regular grids
  - Benchmark problems
- **Conclusion**

Equation of motion

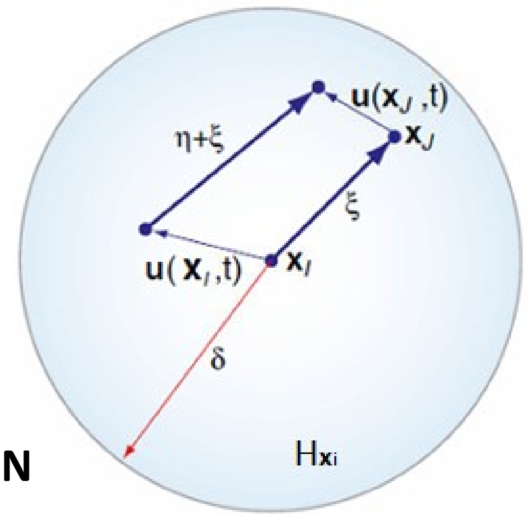
- For a given body R_0, the equation of motion of an infinitesimal particle of material is defined by means of the following expression:

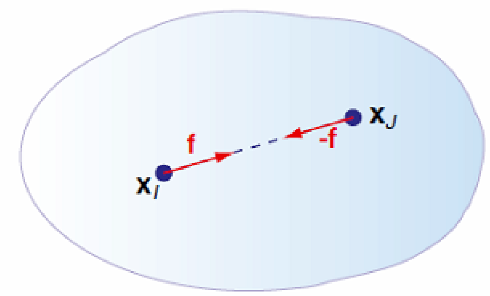
$$\rho\ddot{\boldsymbol{u}}(\boldsymbol{x}_i,t) = \nabla[\sigma] + \boldsymbol{b}(\boldsymbol{x}_i,t)$$

$$\rho\ddot{\boldsymbol{u}}(\boldsymbol{x}_i,t) = \int_{H_{\boldsymbol{x}_i}} \boldsymbol{f}\big(\boldsymbol{u}(\boldsymbol{x}_j,t) - \boldsymbol{u}(\boldsymbol{x}_i,t), \boldsymbol{x}_j - \boldsymbol{x}_i\big)dV_j + \boldsymbol{b}(\boldsymbol{x}_i,t)$$

$$H_{\boldsymbol{x}_i} = \{\boldsymbol{x}_j \in R_0 : \|\boldsymbol{x}_j - \boldsymbol{x}_i\| \leq \delta\}$$ → **HORIZON**

$$\boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = f(\eta,\xi)\frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\|\boldsymbol{\xi}+\boldsymbol{\eta}\|}$$ → **PAIRWISE FORCE FUNCTION**





$\boldsymbol{u}$ → displacement vector field
ρ → mass density
$\boldsymbol{b}$ → body force density
$\boldsymbol{\xi} = \boldsymbol{x}_j - \boldsymbol{x}_i$ → initial relative position
$\boldsymbol{\eta} = \boldsymbol{u}_j - \boldsymbol{u}_i$ → relative displacement

Images by W. Liu and J.W. Honget

Constitutive law

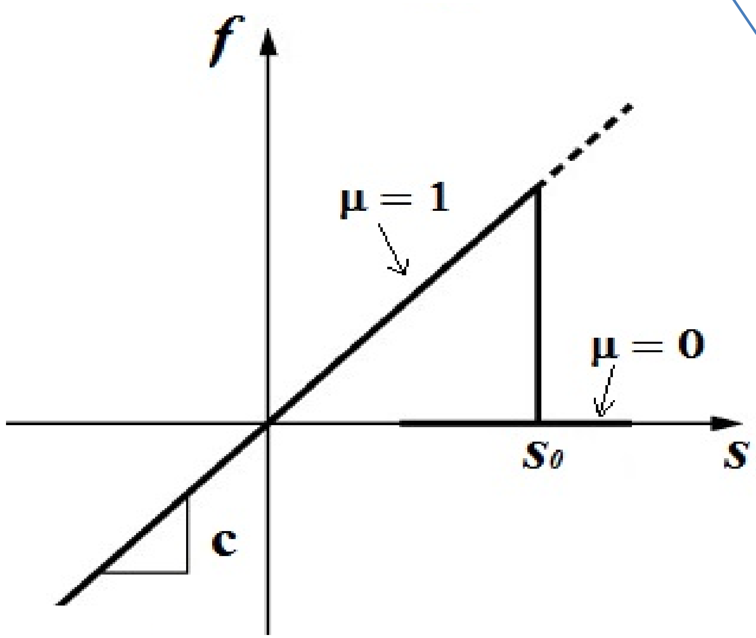
- A brittle elastic material is modeled adopting the constitutive law called PMB (**Prototype Microelastic Brittle**). The scalar pairwise force function takes the form:

$$f(\eta, \xi) = \mu(\xi) \cdot c \cdot s$$

$$s = \frac{\|\boldsymbol{\xi} + \boldsymbol{\eta}\| - \|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|}$$ **stretch of the bond**

$$c \propto \frac{E}{\delta^3}$$ **micromodulus**

$$\mu(\xi) = \begin{cases} 1 & if\ s < s_0 \propto \sqrt{G_0/_{E\delta}} \\ 0 & otherwise \end{cases}$$



E:= Young's modulus
G_0 := Fracture energy

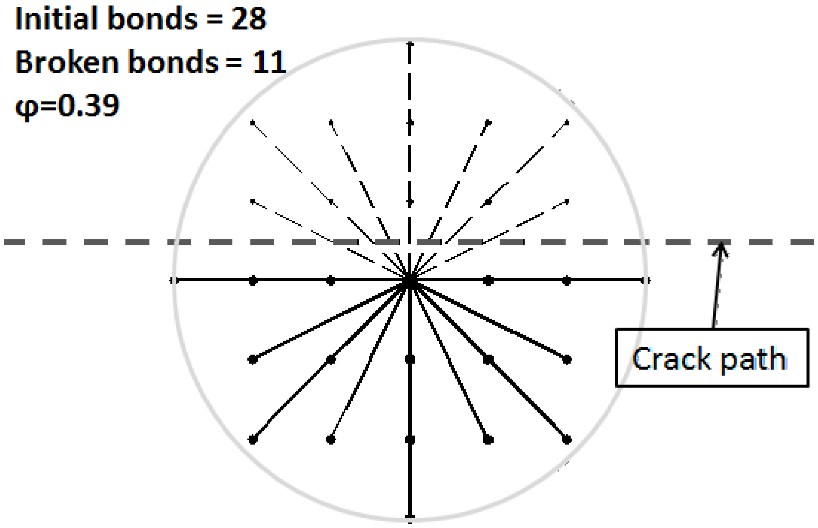
Damage definition

- It is possible to define a non-ambiguous state of **material damage** at every point $\boldsymbol{x}_i$ of the body as:

$$\varphi(\boldsymbol{x}_i) = \frac{broken\ bonds}{initial\ bonds}$$

$\phi(\boldsymbol{x}_i)$ = 0 means **undamaged state**

$\phi(\boldsymbol{x}_i)$ = 1 means **all broken bonds**



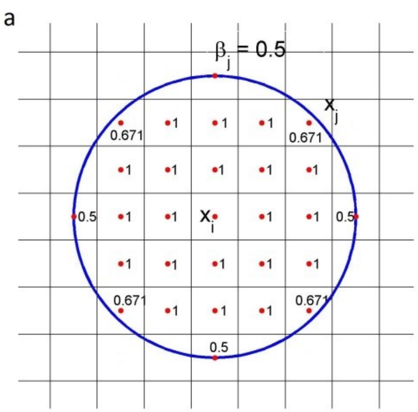
Images by ing. M. Duzzi

Some remarks about the numerical discretization

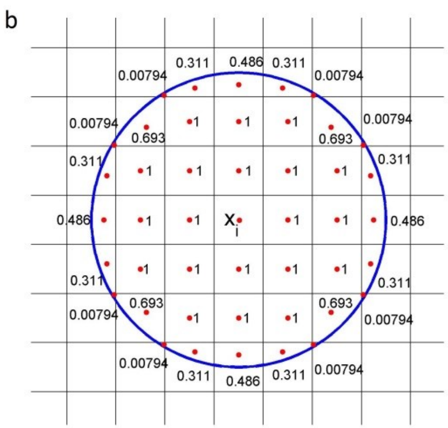
- Mesh-free method with a uniform structured grid and the Gauss quadrature mid-point space integration solved through the Velocity-Verlet explicit scheme:

$$\rho \ddot{\boldsymbol{u}}_i^n = \sum_j \boldsymbol{f}(\boldsymbol{u}_j^n - \boldsymbol{u}_i^n, \boldsymbol{x}_j - \boldsymbol{x}_i) V_j \beta_j + \boldsymbol{b}_i^n, \qquad \forall\, \boldsymbol{x}_j \in H_{x_i}$$

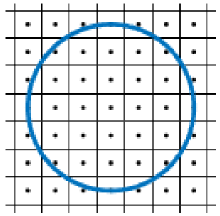
$$\rho \ddot{\boldsymbol{u}}_i^n = \sum_k \boldsymbol{f}(\boldsymbol{u}_k^n - \boldsymbol{u}_i^n, \boldsymbol{x}_k - \boldsymbol{x}_i')\, \beta_i \Delta V_k - \sum_j \boldsymbol{f}(\boldsymbol{u}_j^n - \boldsymbol{u}_i^n, \boldsymbol{x}_j - \boldsymbol{x}_i) \beta_j \Delta V_j + \boldsymbol{b}_i^n, \forall\, \boldsymbol{x}_k \in H'_{x_i}, \forall\, \boldsymbol{x}_j \in H_{x_i}$$



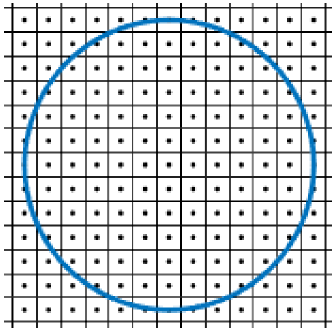
Partial Area - PDLAMMPS algorithm



Improved Partial Area Hybrid



δ – convergence
(δ↘, *m*=constant)



***m* – convergence**
(m↗, δ =constant)

m ratio=δ/Δx

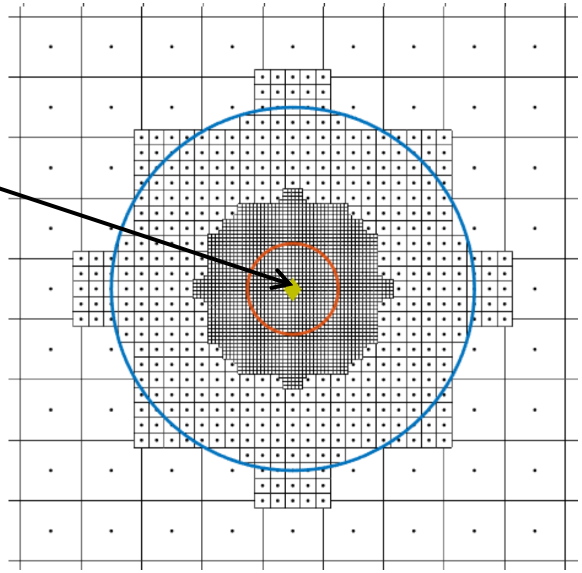
AGRS

- The adaptive refinement and scaling allows to reduce in automatic mode (by means of a trigger) both the grid spacing and the horizon only in the regions of interest, as in the proximity of the crack tips during their propagation :

Selected coarse node by trigger

$\boldsymbol{\delta}$ – convergence

(δ↘, m=constant)



old horizon

new horizon

➢ based on potential energy (suggested in literature)

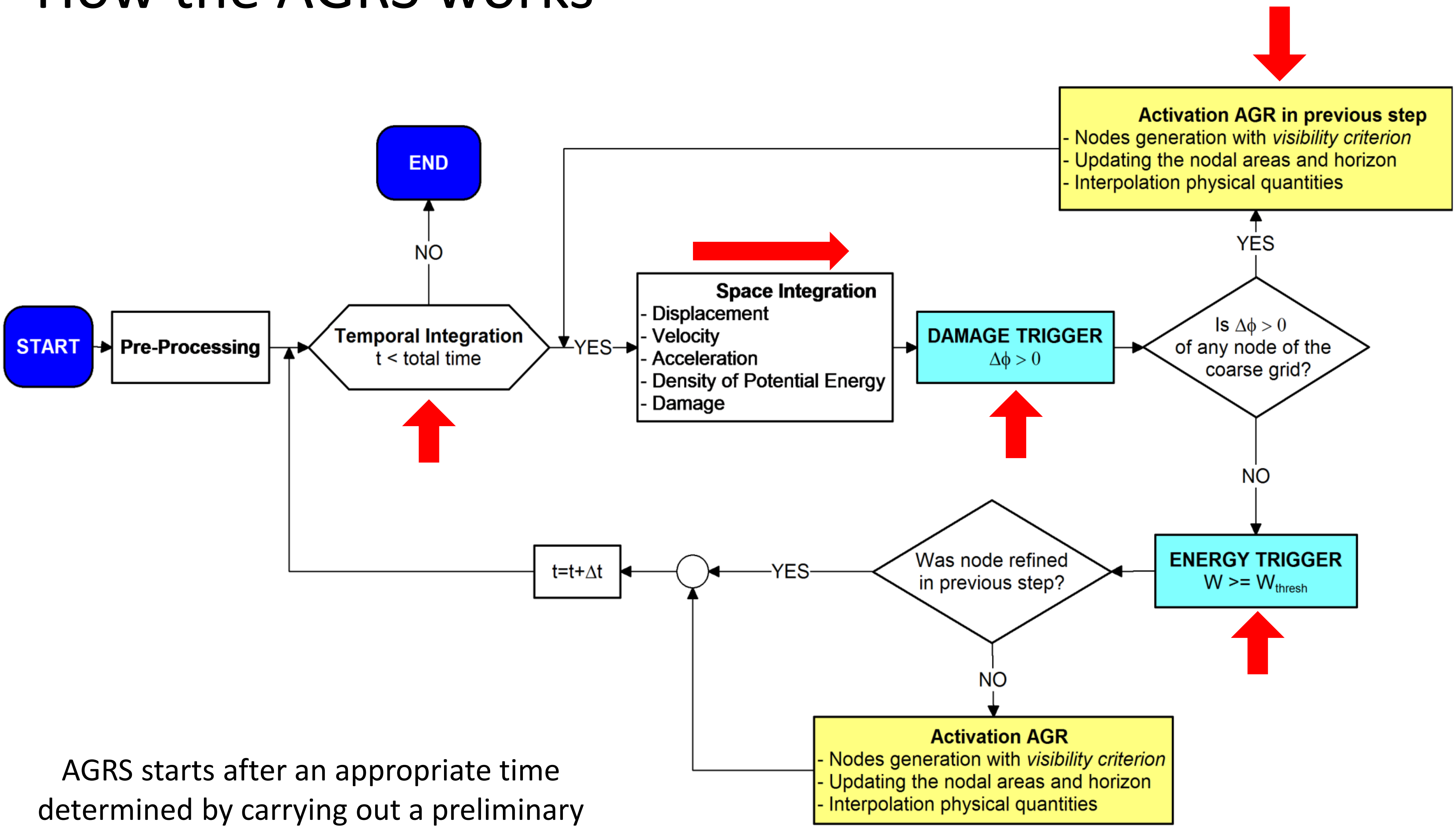
$$W(\boldsymbol{x}_i) \geq W_{threshold}$$

➢ based on damage state, introduced by us

$$\Delta\phi(\boldsymbol{x}_i) = \phi(\boldsymbol{x}_i) - \phi_0(\boldsymbol{x}_i) > 0$$

$\phi_0(\boldsymbol{x}_i)$ is the initial damage state of the nodes

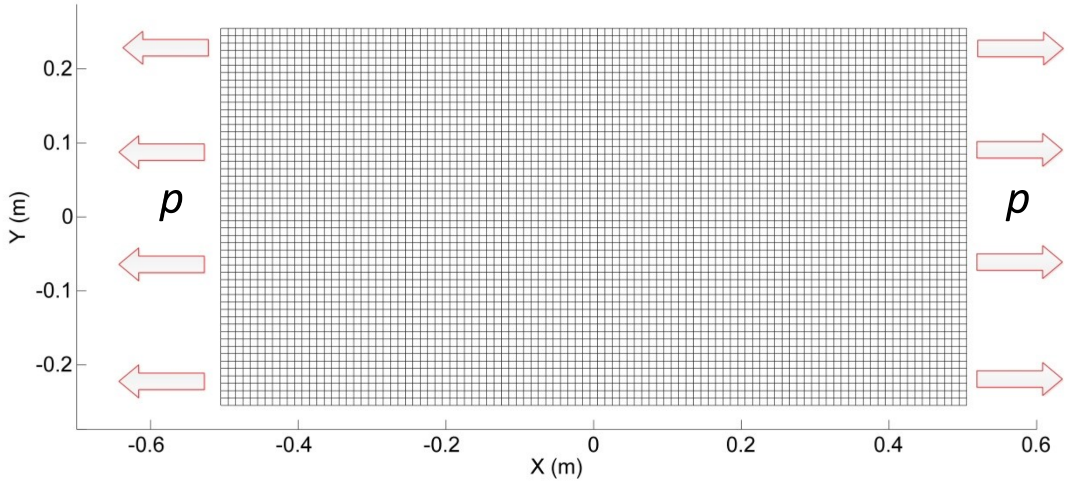
How the AGRS works



AGRS starts after an appropriate time determined by carrying out a preliminary analysis on the coarse grid

Static Test

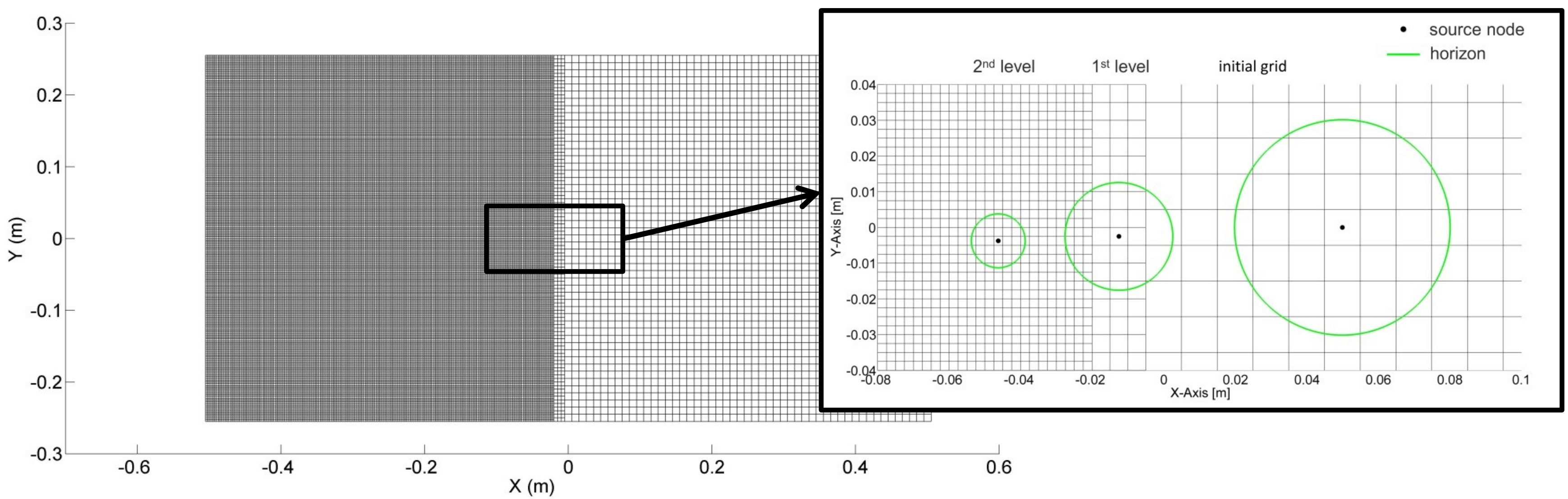
- A 2D static elastic linear problem is addressed through the comparison of the numerical peridynamic solution with the analytical solution of classic theory of mechanics:



$$u_X = (X, Y = 0) = \frac{p}{E} X$$

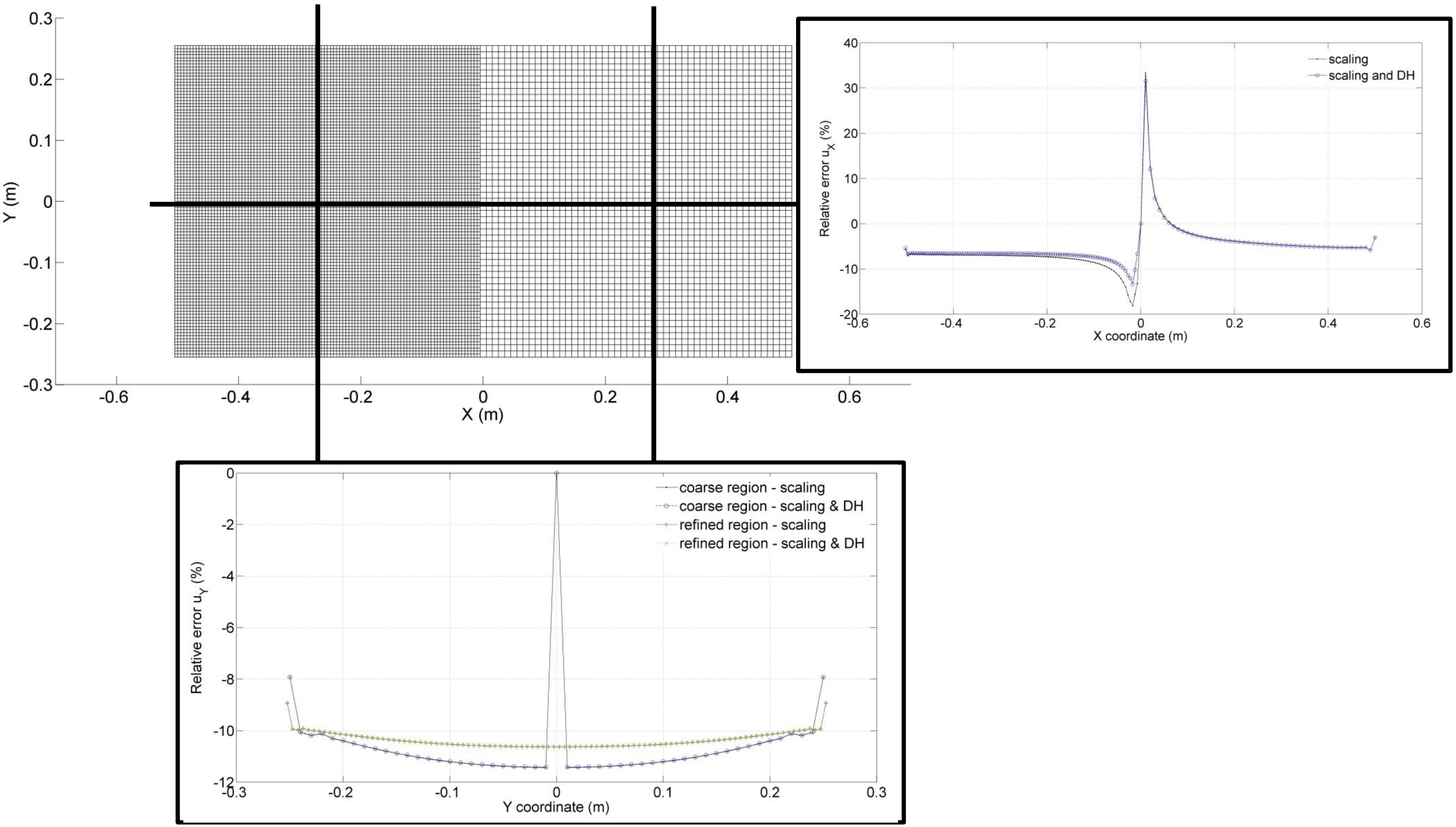
$$u_y = (X = 0, Y) = -\upsilon \frac{p}{E} Y$$

- Example of 2nd level of refinement and scaling applied:



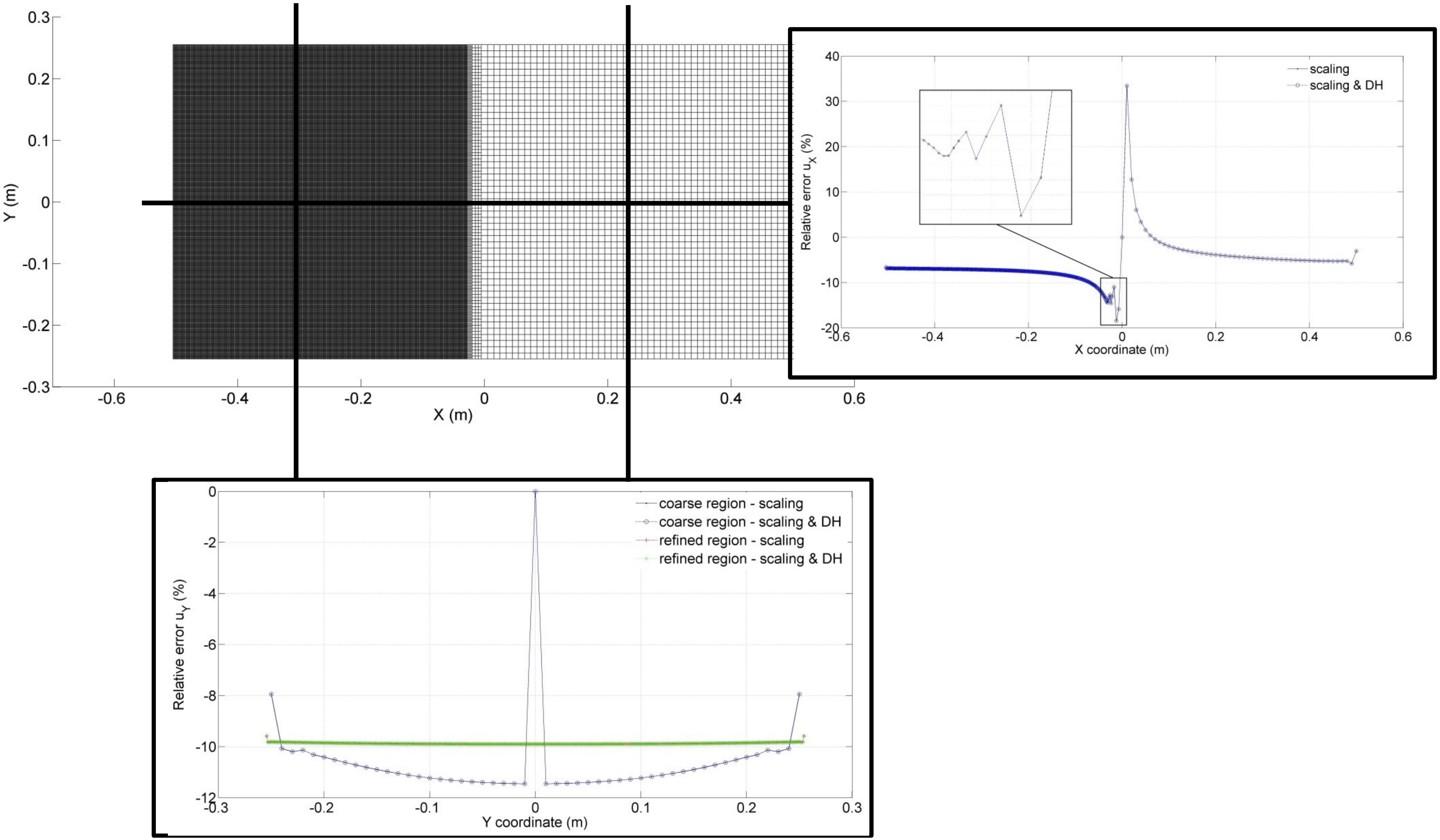
Static Test

- The PD solution between Scaling and Scaling & Dual-horizon formulation are compared for the 1st level of refinement/scaling:



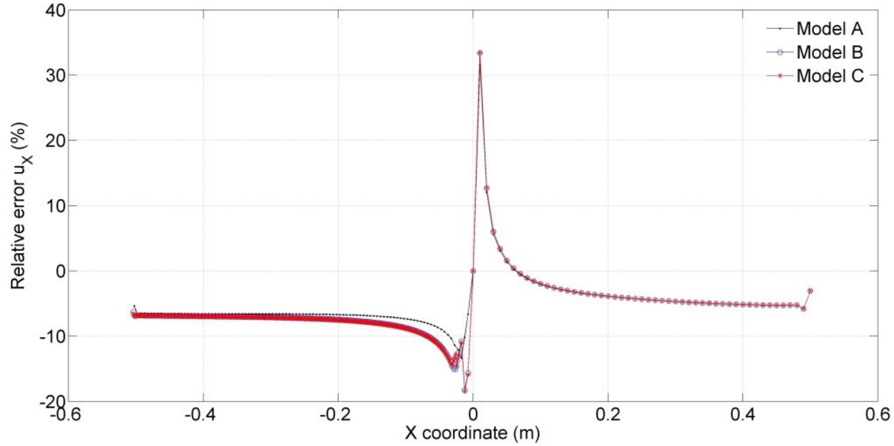
Static Test

- The PD solution between Scaling and Scaling & Dual-horizon formulation are compared for the 3rd level of refinement/scaling:



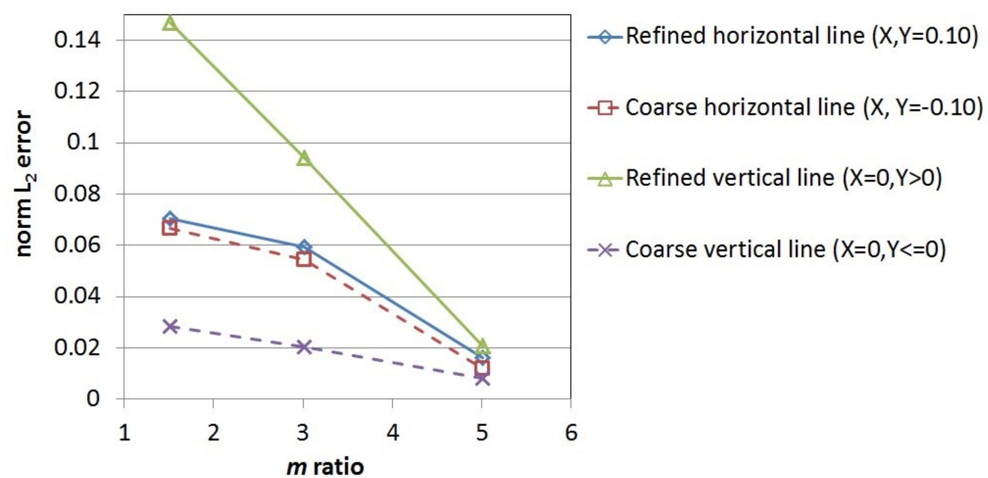
Static Test

- The comparison between the solutions obtained by applying different levels of refinement/scaling highlights that higher levels of refinement do not affect the result:



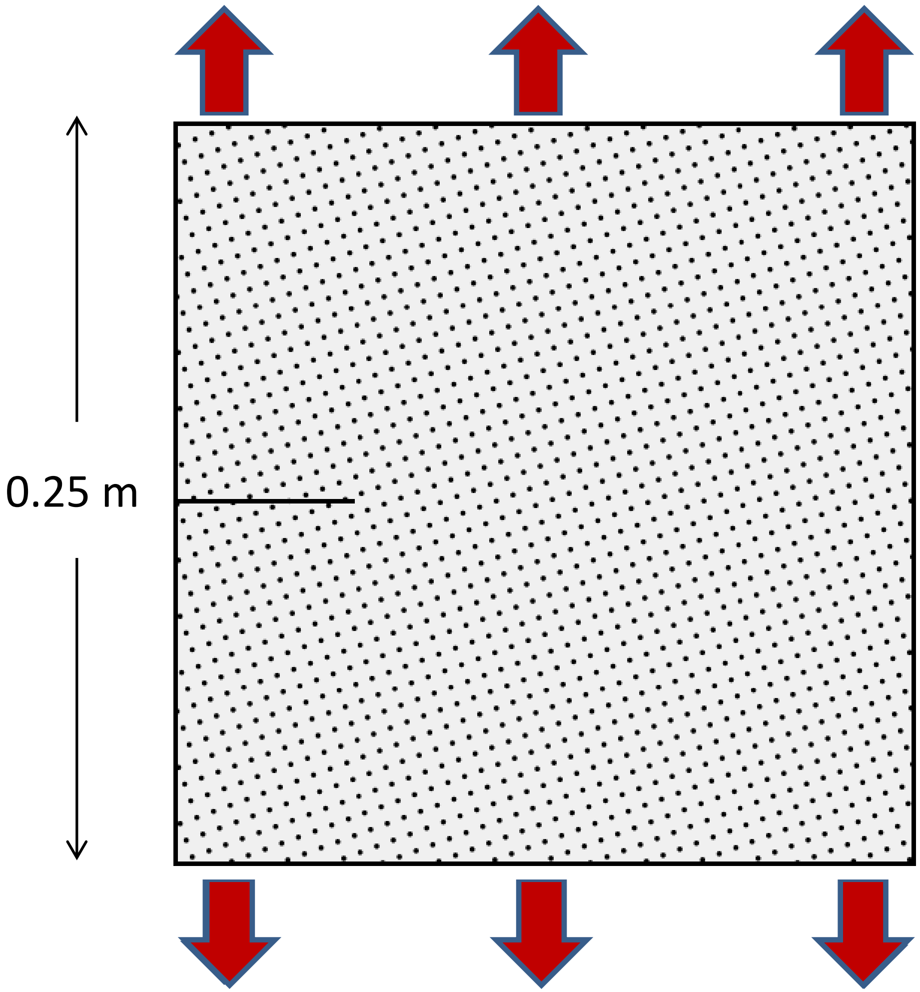
- The convergence study on *m* ratio shows as the rate of convergence of the refined region is higher than the coarse one:

$$L_2\ error = \frac{1}{q}\frac{\sqrt{\sum_{i=1}^{j}(u_i^{PD} - u_i^{analitycal})^2}}{\sqrt{\sum_{i=1}^{j}(\,u_i^{PD}\,)^2}}$$



Addressing grid sensitivity in peridynamics

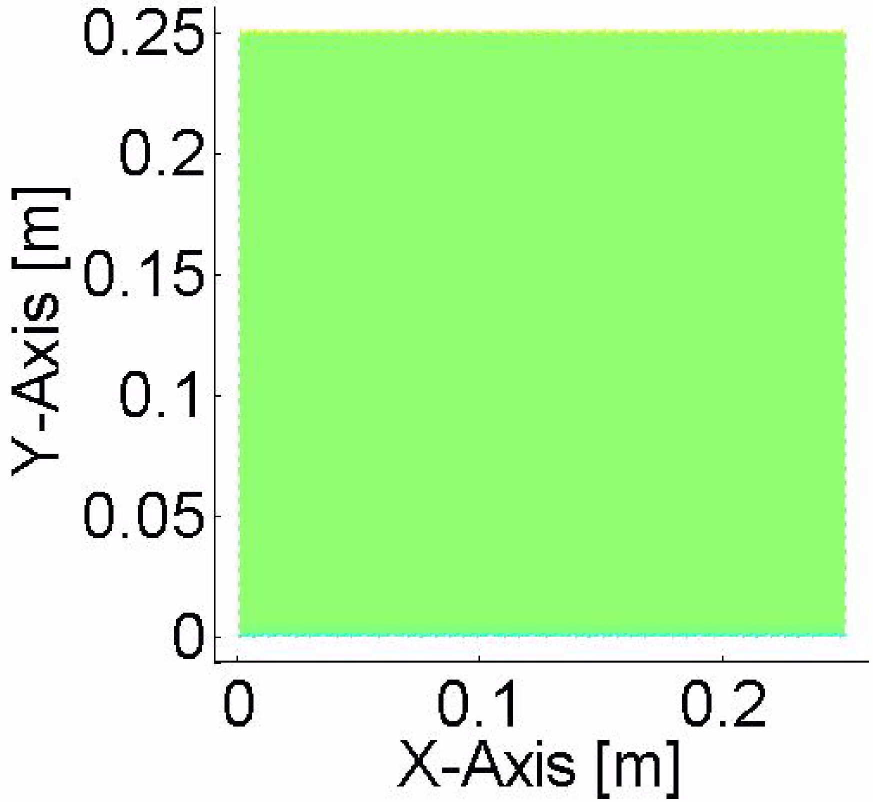
- Problem of a 2D pre-cracked square plate subjected to a traction load:



- **The grid is rotated with rispect to the direction of pre-crack line**
- **Quasi-Static and Dynamic analysis**

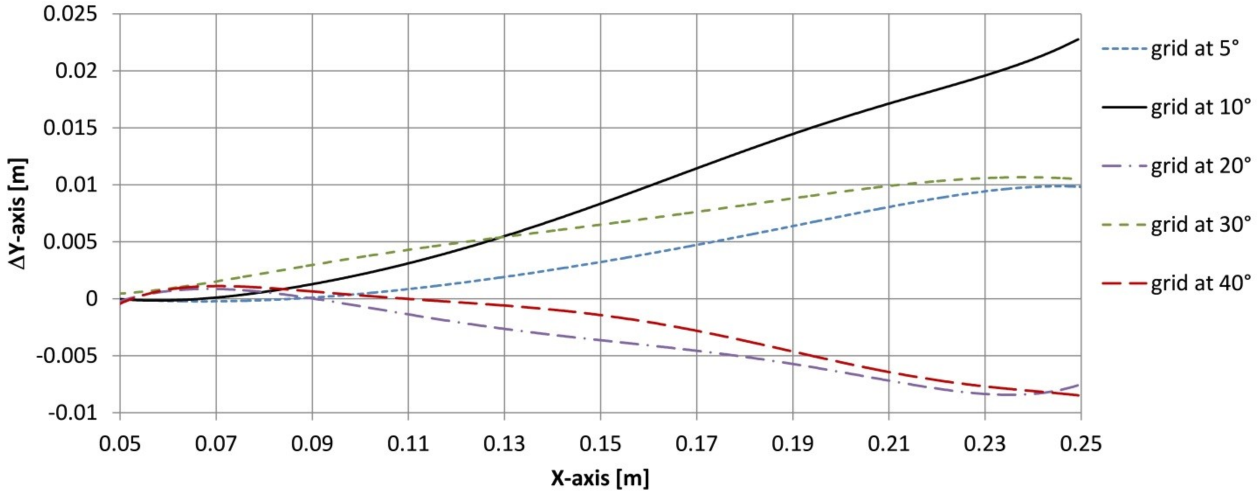
Addressing grid sensitivity in peridynamics

- Waves propagation regarding the dynamic case of the grid rotated of 10° with m = 3:

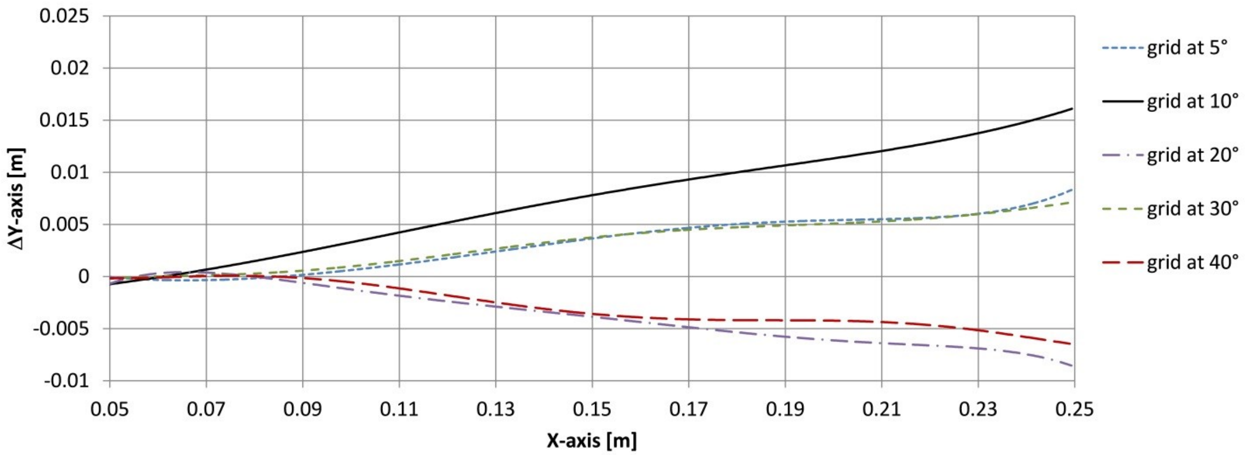


Addressing grid sensitivity in peridynamics

- Crack paths obtained with different rotated grids with *m* = 3, dynamic cases:



- Crack paths obtained with different rotated grids with *m* = 3, quasi-static cases:

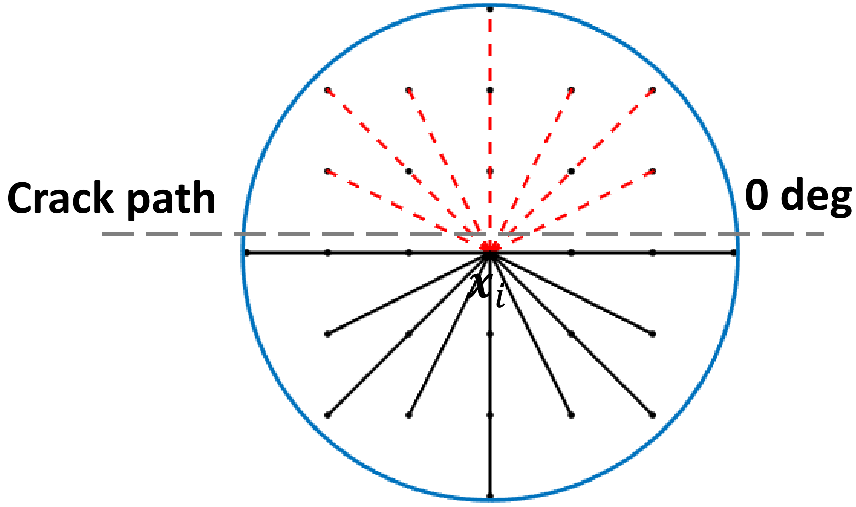


Numerical explanation of grid sensitivity

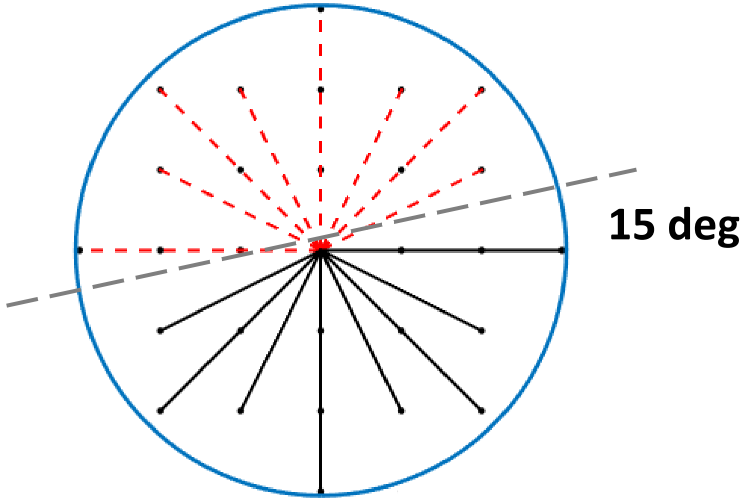
- This type of space discretization introduces an anisotropy on the damage state of the node, namely an anisotropy on the energy required to break the bonds along a specific direction:

$$\phi = \frac{broken\ bonds}{initial\ bonds}$$

Example with *m* ratio = 3



$$\phi_0 = 0.38$$

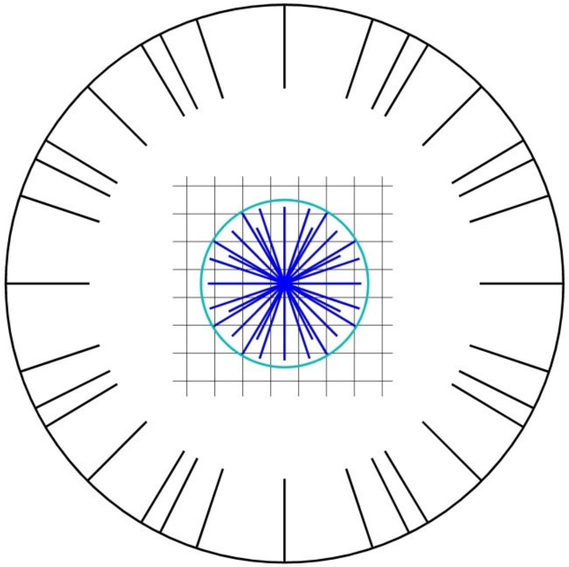


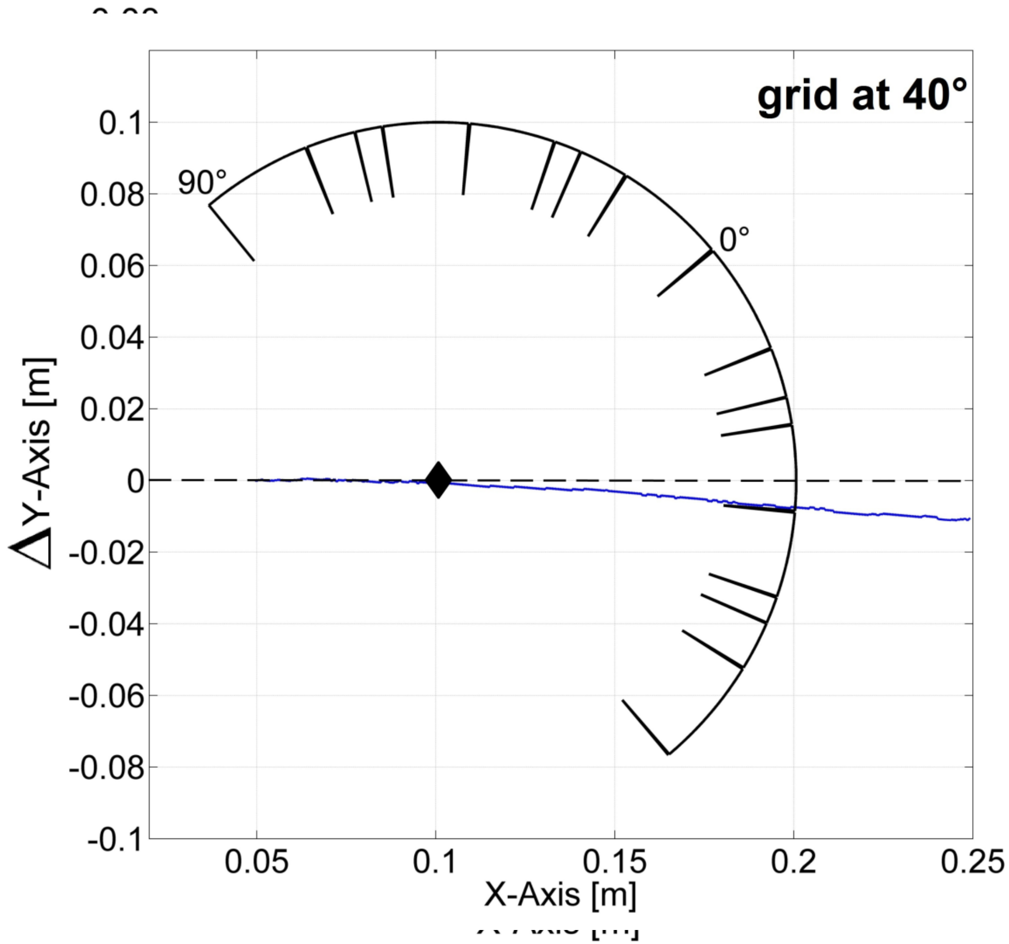
$$\phi_{15} = 0.48$$

Numerical explanation of grid sensitivity

- The directions of minimum energy required to break the bonds along a specific direction match the directions of the bonds:

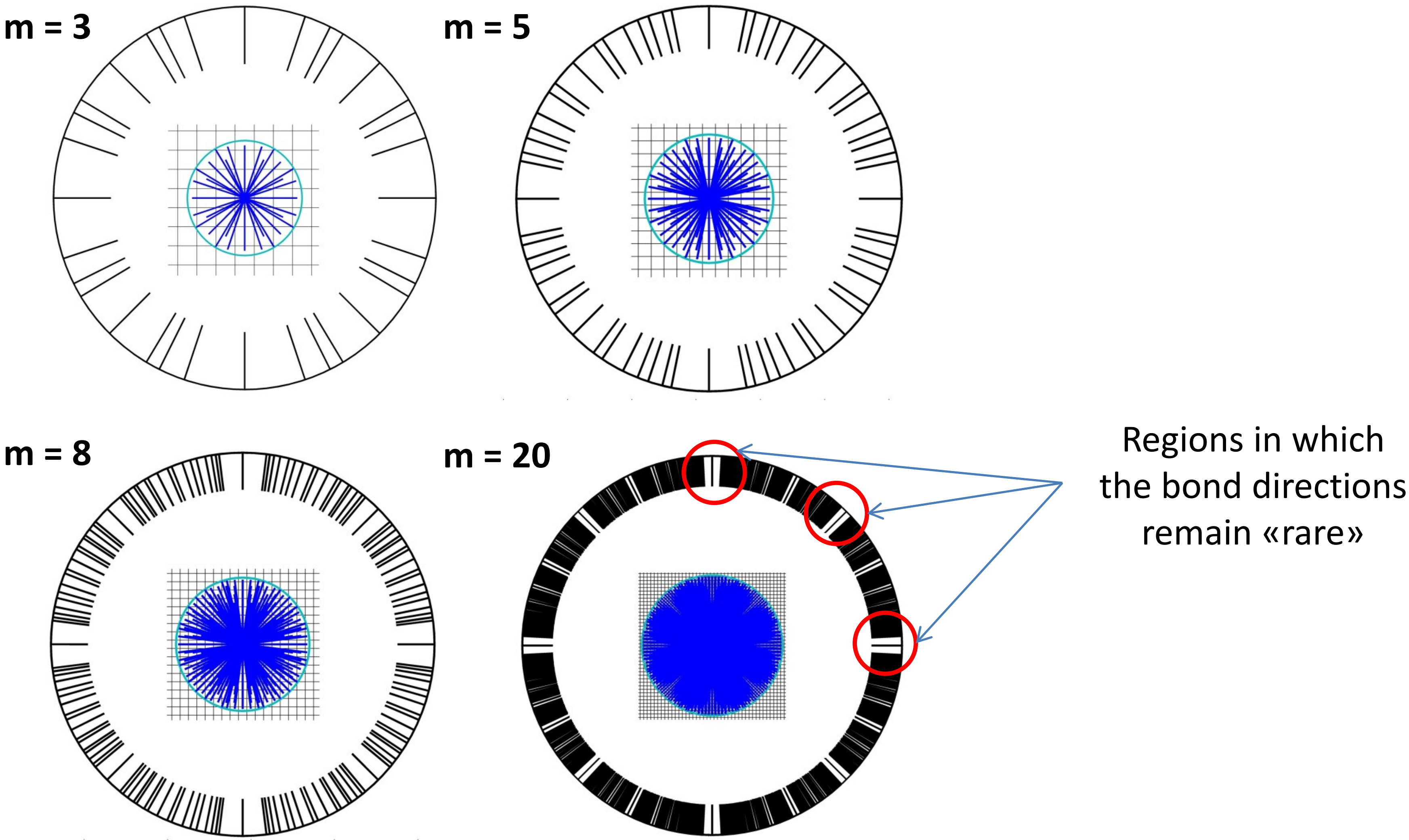
Example with *m* ratio = 3





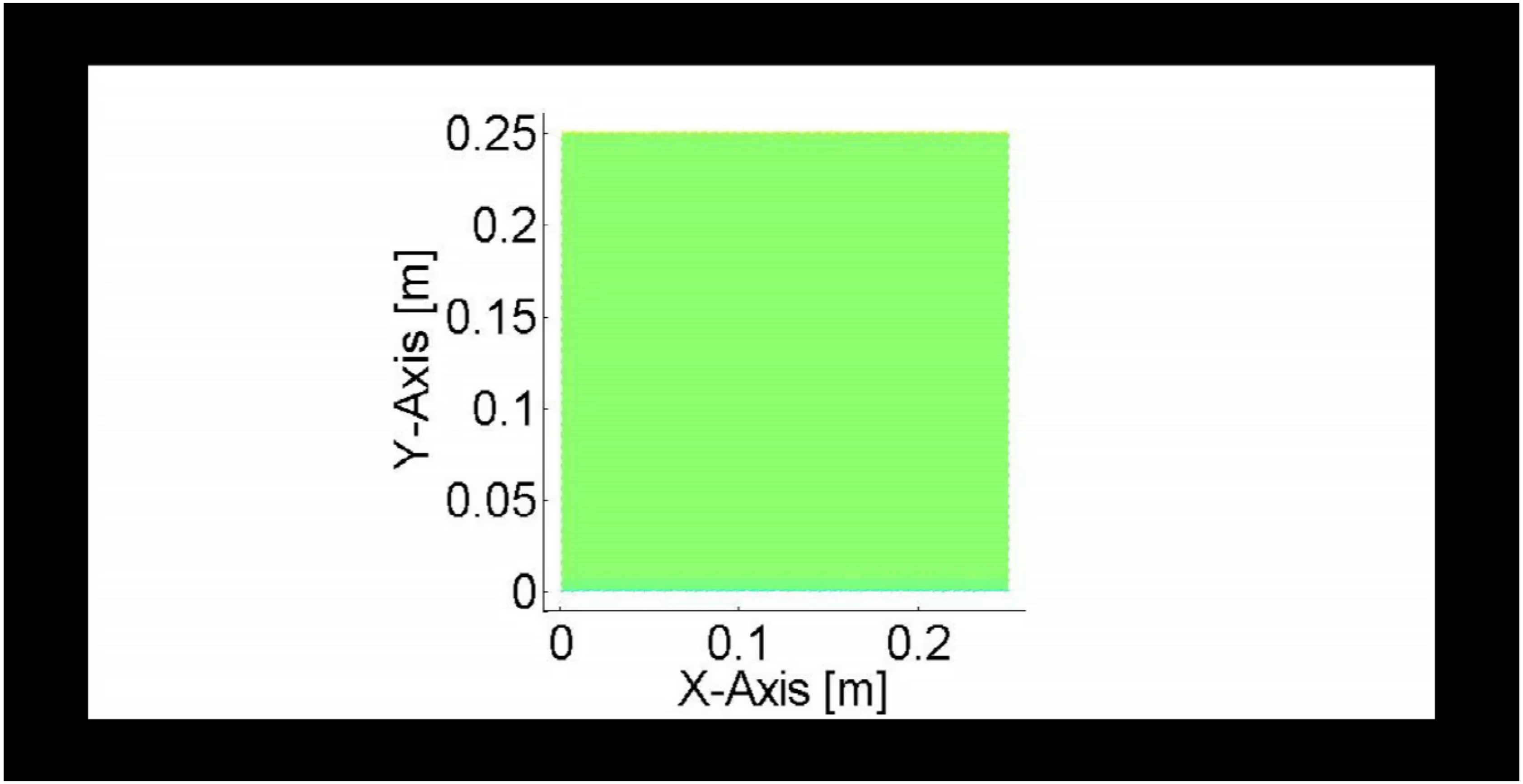
Numerical explanation of grid sensitivity

- When the *m* ratio increases both the number of the minimum energy directions increases and the gap energy between them and the other directions reduces:



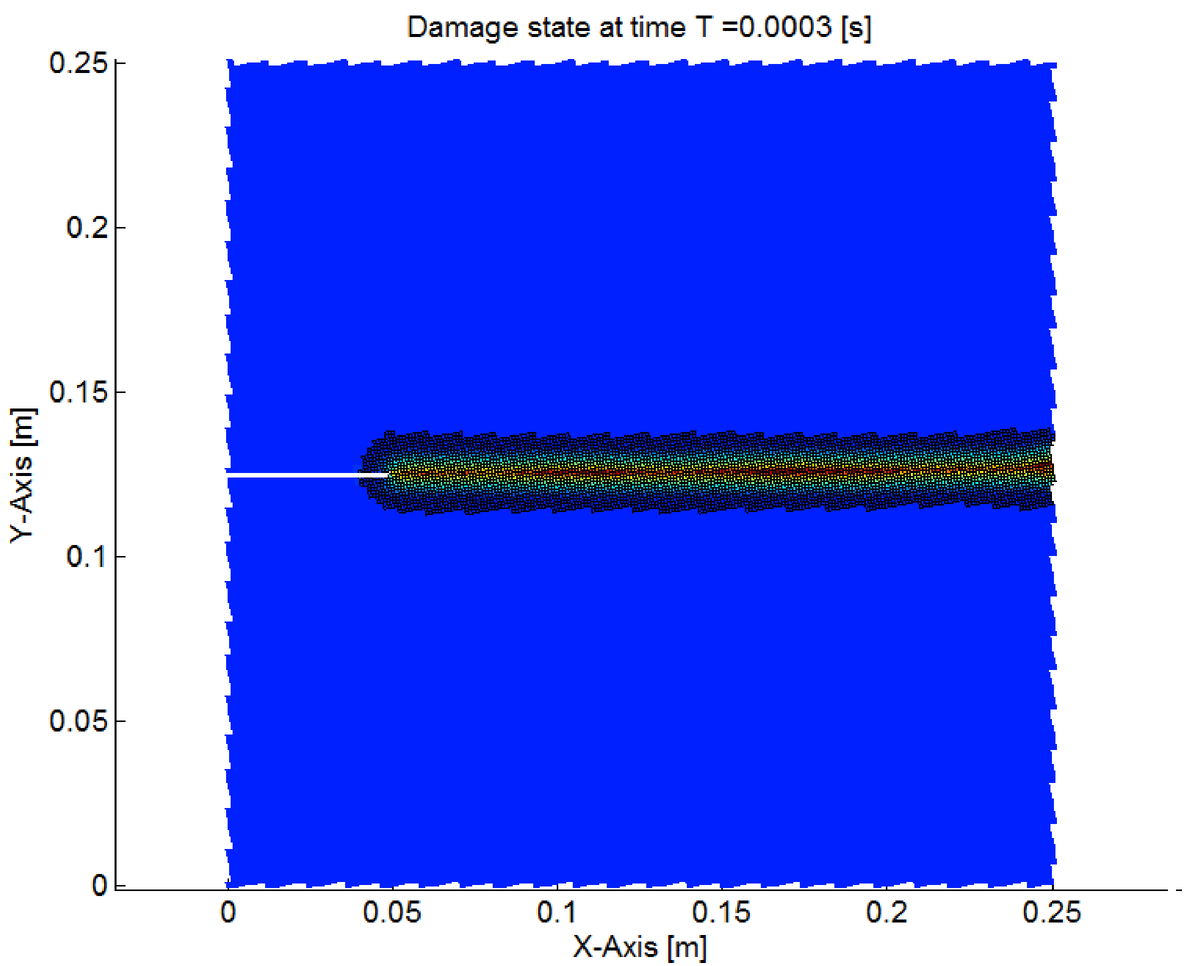
Addressing grid sensitivity in peridynamics

- With reference to the worst case of grid rotated of 10 degree, it is possible to see as an increase of *m* ratio from 3 to 6-7 is enough to eliminate the dependence of crack propagation on grid orientation:



Addressing grid sensitivity with AGRS

- Application of the 1st level of AGRS when the grid is rotated of 10°, the **horizon is kept constant**:



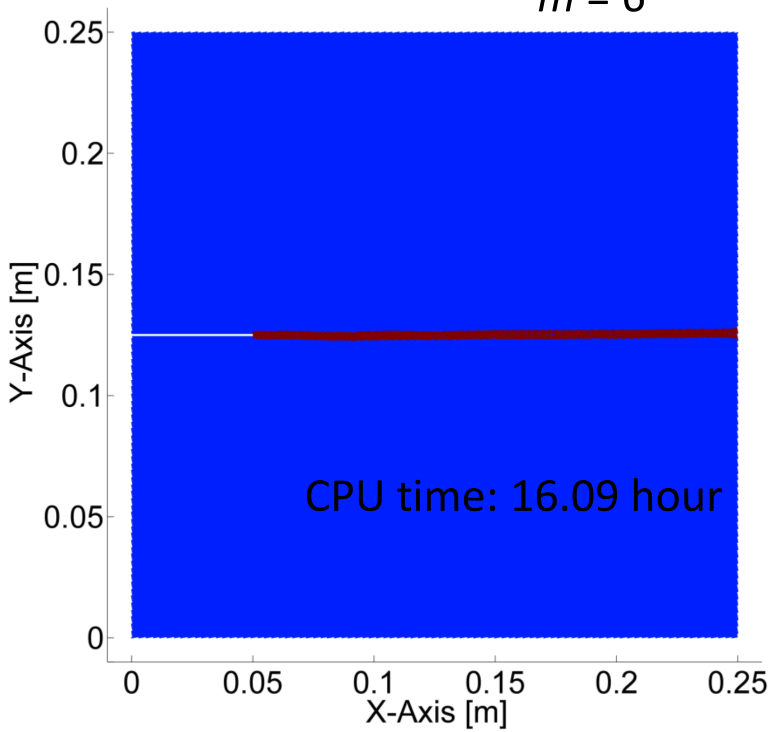
- **Initial Grid :** 15,600 nodes
  $\Delta x = 2\ mm$
  $m = 3$

- **Refined region:** $\Delta x = 1\ mm$
  $m = 6$

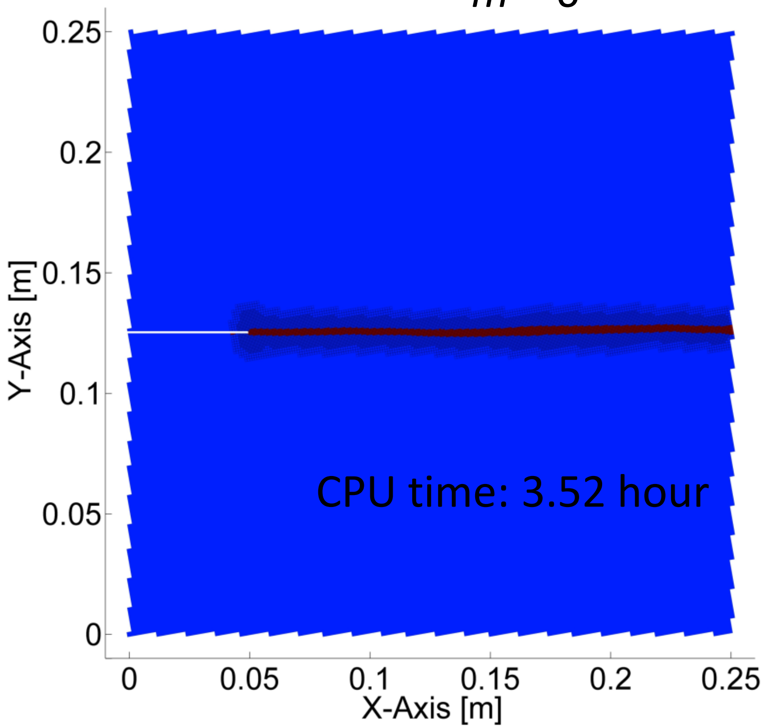
Addressing grid sensitivity with AGRS

- Application of the 2nd level of AGRS when the grid is rotated of 10°, the **horizon shrinks**:

- **Uniform Refined region:** $\Delta x = 0.5\ mm$, $m = 6$

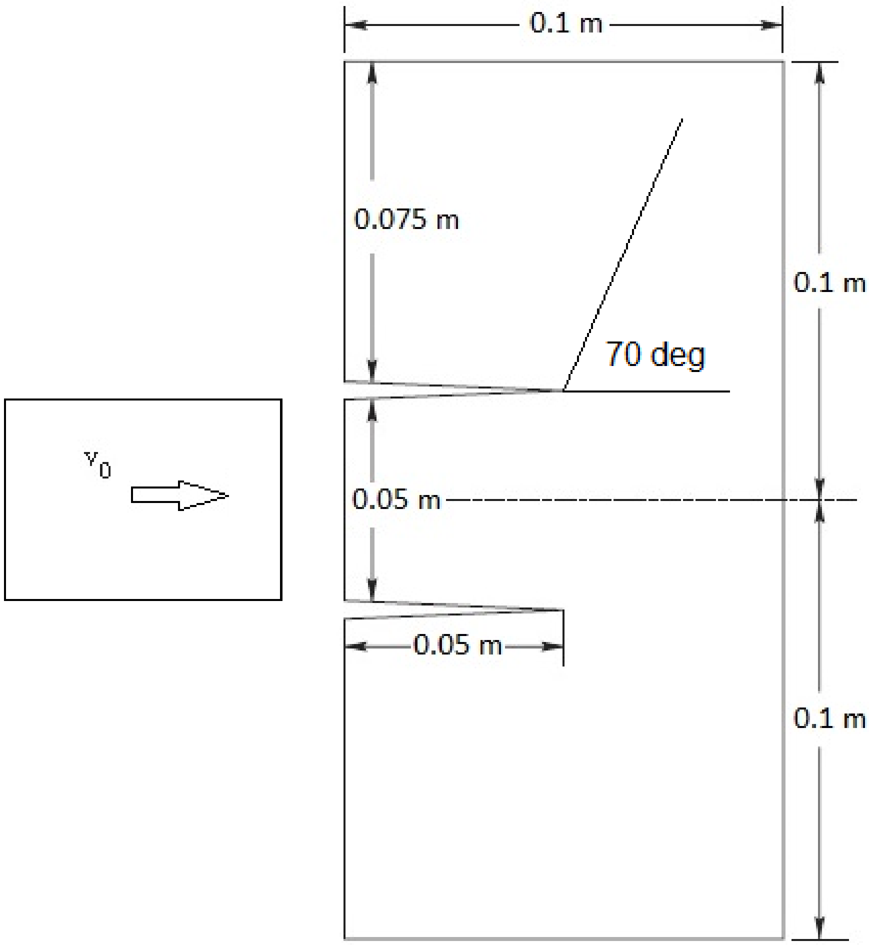


- **Initial Grid :** 15,600 nodes
  $\Delta x = 2\ mm$
  $m = 3$
- **Refined region:** $\Delta x = 0.5\ mm$, $m = 6$



Benchmark problem: Kalthoff-Winkler's experiment

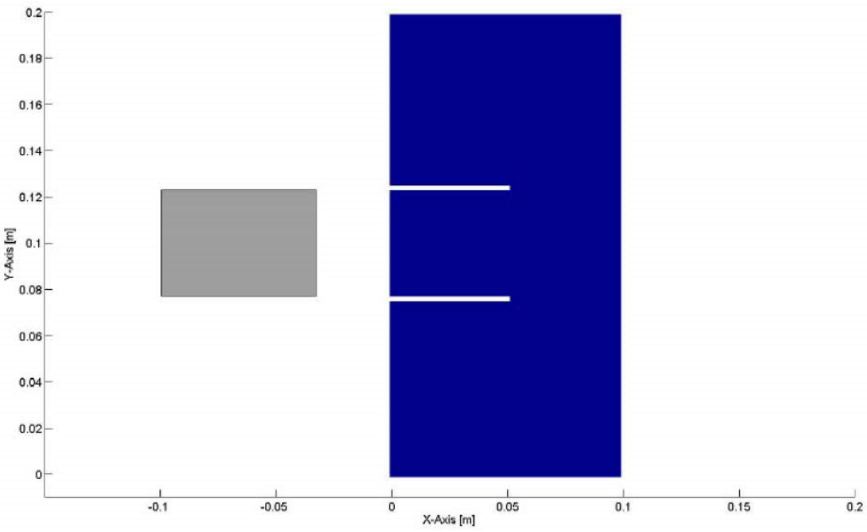
- Setup experiment:
  - **Material 18Ni1900**: $E = 190\ GPa$, $\rho = 8000\ kg/m^3$, $G_0 = 22170\ J/m^2$
  - **Simulation** : $t_{tot} = 52\ \mu s$ ($\Delta t_{min} = 20\ ns$)
  - **Initial Grid** : 5,000 nodes, $\Delta x = 2\ mm$, $\delta = 6\ mm$
  - **Energy Trigger** : $W \geq 0.7\ W_{max}$
  - **Damage Trigger** : $\Delta\phi \geq 0$
  - **δ-convergence** (m=3)



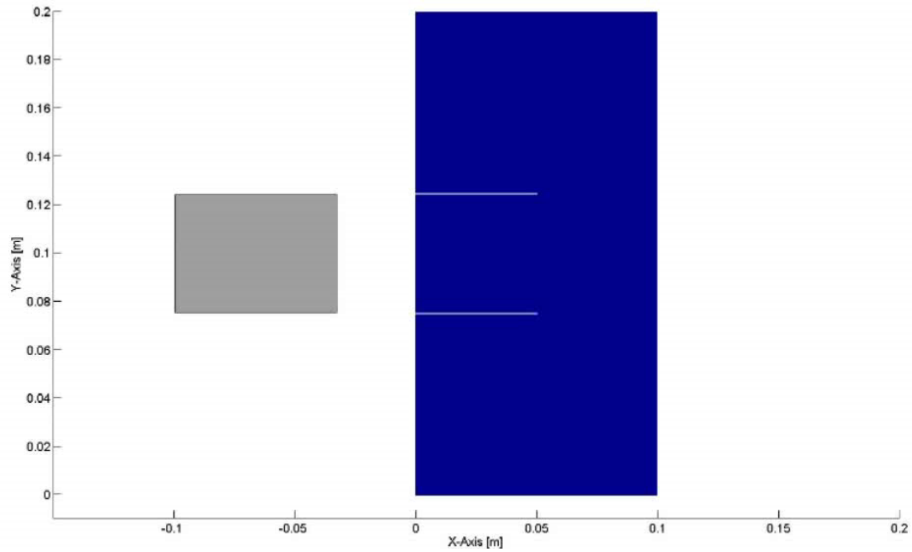
Kalthoff J (2000) Modes of dynamic shear failure in solids. Int J Frac 101:1–31

Benchmark problem: Kalthoff-Winkler's experiment

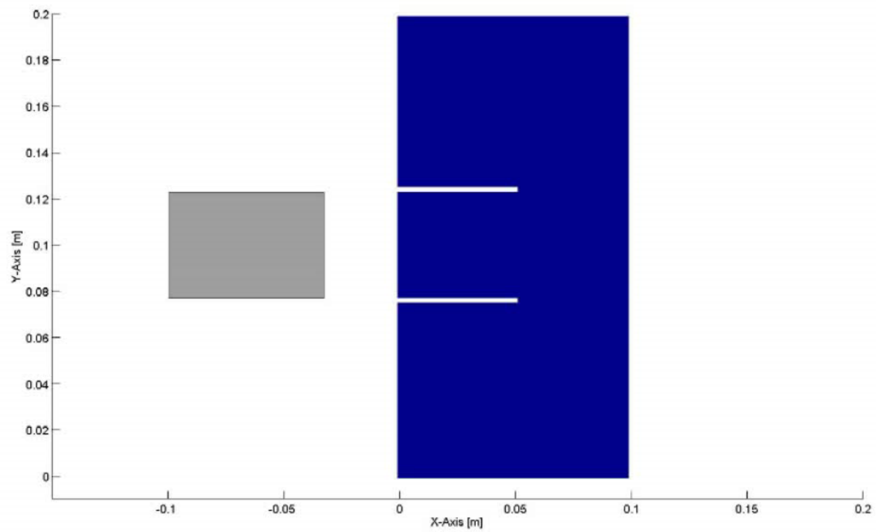
Uniform coarse grid ($\Delta x = 2mm$)



Uniform refined grid ($\Delta x = 0.5mm$)



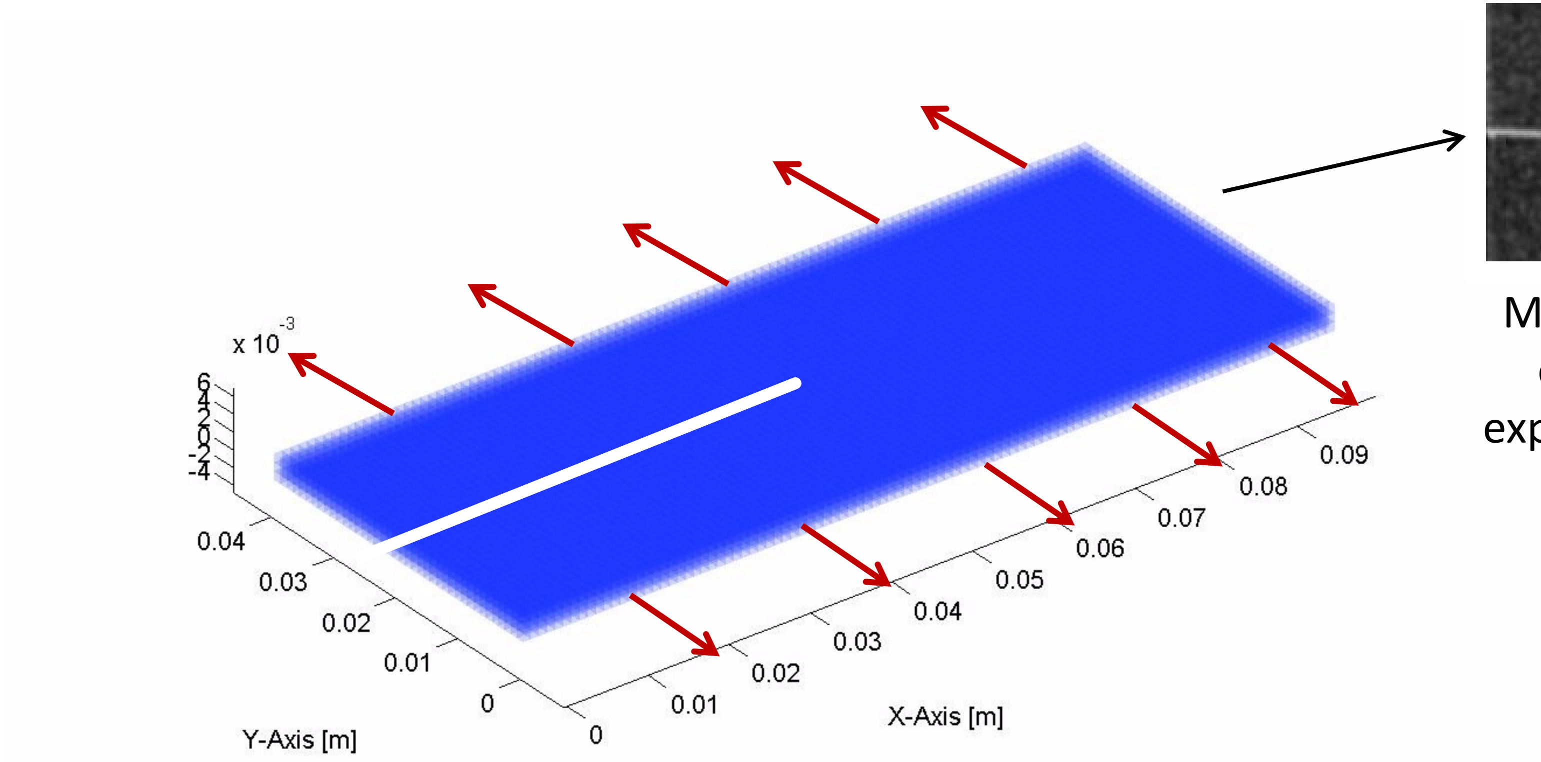
Adaptive grid with 2nd level of refinement
($\Delta x_0 = 2mm$, $\Delta x_2 = 0.5mm$)

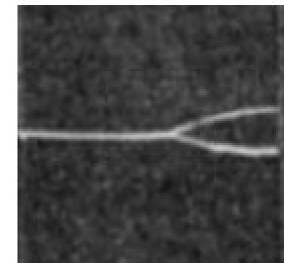


Adaptive model is able to capture the right angle of approximately 70°

3D Adaptive grid refinement/scaling

- Crack branching of pre-cracked glass plate under traction, the 1st level of AGRS is applied in a 3D model:



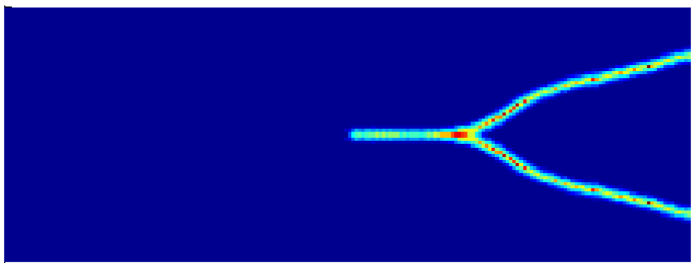


Morphology observed expermentally

Activities related to my Ph.D

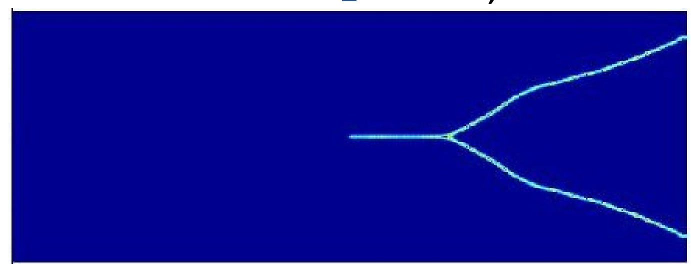
- Optimization of the pre-existent Matlab codes in the context of dynamic simulations with tools such as <u>MEX files</u>:

16,000 nodes



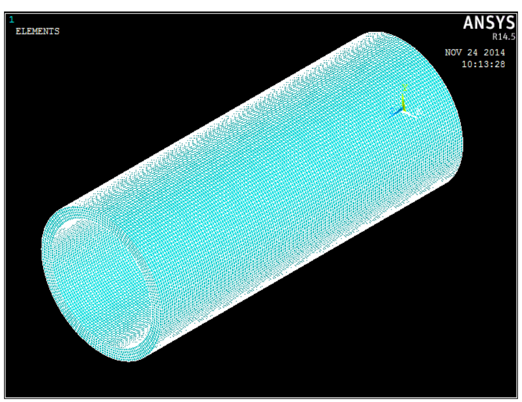
≈ **9 hours**

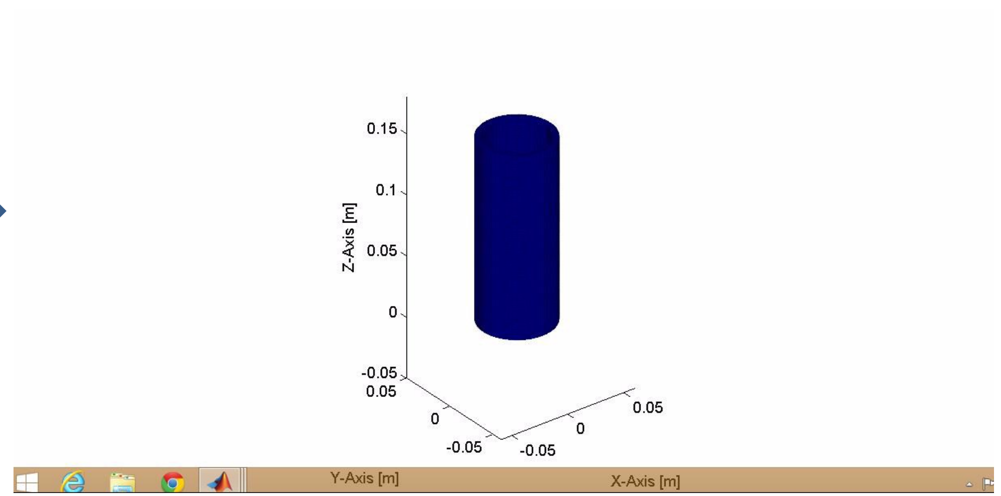
120,000 nodes



≈ **5 hours**

- Implementation of codes to import a general 3D mesh in Matlab environment





Publications

Dipasquale D., Zaccariotto M. and Galvanetto U. (2014) *Crack propagation with adaptive grid refinement in 2D peridynamics*. International Journal of Fracture, Vol 190, Issue (1), pp 1-22.

Dipasquale D., Sarego G., Zaccariotto M., Galvanetto U. (2014) *Peridynamics with adaptive grid refinement*. Proceeding of the 11th World Congress on Computational Mechanics (WCCM), Spain

Dipasquale D., Zaccariotto M., Sarego G., Duzzi M., Galvanetto U. (2014) *Peridynamics computations with variable grid size*. Proceeding of the 27th Nordic Seminar on Computational Mechanics (NSCM-27), Sweden

Galvanetto U., Zaccariotto M., **Dipasquale D.**, Sarego G., Duzzi M. (2014), *Grid refinement in peridynamic computational applications*. Contribution of International CAE Conference, Italy.

Dipasquale D., Zaccariotto M., Duzzi M., Galvanetto U. (2014), Sarego G., *Dynamic and static simulations with peridynamic approach using finite element analysis*. Poster presented at International CAE Conference

Duzzi M., Zaccariotto M., **Dipasquale D.**, Galvanetto U. (2014), *A Concurrent Multiscale Model to Predict Crack Propagation in Nanocomposite Materials with the Peridynamic Theory*. Poster will be presented at International NanotechItaly2014 Conference, Italy

Dipasquale D., Sarego G., Zaccariotto M., Galvanetto U. (2015), Dependence of crack paths on the orientation of regular peridynamic grids, Eng. Fract. Mech., Vol. 160, pp. 248-263

Galvanetto U., Zaccariotto M., **Dipasquale D.**, Sarego G. (2015), *Enhanced 2D lamina formulation for composite materials, simulation with a peridynamics approach*, Abstract In: International Conference on Advances in Composite Materials and Structures, Book of Abstracts, pp. 60-61, Instambul, Turkey, 13-15 April 2015

Publications

Dipasquale D., Sarego G., Shojaei A., Zaccariotto M., Galvanetto U., (2015) *Addresing grid sensitivity in Peridynamics: an adaptive refinement approach*. Abstract In: International Conference on Computational Modelling of Fracture and Failure, pp. 248-249, Cachan , France, 3-5 June

Zaccariotto M., Sarego G., **Dipasquale D.**, Galvanetto U. (2015), *Remarks on constitutive laws and influence functions used in the Peridynamic theory*. Abstract In: International Conference on Computational Modelling of Fracture and Failure, pp. 248-249, Cachan, France, 3-5 June

Zaccariotto M., Sarego G., **Dipasquale D.,** Shojaei A., Mudric T., Duzzi M., Galvanetto U. (2015), *Discontinuos mechanical problems studied with a peridynamics-based approach*, Proceeding of the 23rd Conference of the Italian Association of Aeronautics and Astronautics (AIDAA-23), pp. (19), Turin

Zaccariotto M., Sarego G., **Dipasquale D.**, Galvanetto U. (2015), Alternative thin plate formulation using a peridynamic approach. Contribution In: SPB 2015 International Conference on Shells, Plates and Beams, pp. 55-56, Bologna

Zaccariotto M., Sarego G., **Dipasquale D.**, Galvanetto U. (2015), Strategies for fatigue damage modeling with peridynamics. Contribution In: Book of Abstracts of the 8th International Congress of Croatian Society of Mechanics, Croatia

Dipasquale D., Oterkus E., Sarego G., Zaccariotto M., Galvanetto U. (2016), *Refinement and scaling effects on peridynamic numerical solutions*, Proceeding to be presented In: ASME 2016 International Mechanical Engineering Congress and Exposition, Phoenix, Arizona (USA), 11-17 November 2016.

Mudric T., Zaccariotto M., **Dipasquale D.**, Galvanetto U. (2016) *How to use FEM codes to solve 3D crack propagation problems with Peridynamics*, submitted in the Aerotecnica, missili e spazio.

Conferences

- Participation at 11th World Congress on Computational Mechanics, Barcelona (20-25/07/2014)
- Participation at International CAE Conference, Pacengo del Garda (27-28/10/2014)
- Participation at 4th International Conference on Computational Modeling of Fracture and Failure of Materials and Structures, Paris (02-05/06/2015)
- I will participate in IMECE International Mechanical Engineering Congress & Exposition, Phoenix, Arizona, USA (11-17/11/2016)

Conclusion

- Development of a robust algorithm to implement AGRS on peridynamics through the introduction of a trigger based on damage state of the nodes
- Development of both 2D/3D codes to implement AGRS with Matlab
- Optimization of the pre-existent Matlab codes in the context of dynamic simulations
- Comparison of different peridynamic formulations (Scaling and Dual-horizon) by means of both static and dynamic analysis
- Addressing dependence of crack propagation on grid orientation
- Validation of the numerical results obtained with other methods/experimental results

THANK YOU FOR YOUR ATTENTION

http://www.aerospacestructuresinpadova.org